

UNIVERSITY OF REGINA  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
Math122–001 Linear Algebra I  
*Checklist for Test 1*

## 1 Definitions you should know

1. The *norm* of a vector (i.e. if  $\mathbf{v} = (v_1, v_2, \dots, v_n)$  then  $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$ ).
2. What a *normalized* vector is (for a non-zero vector  $\mathbf{u}$  it is  $\frac{1}{\|\mathbf{u}\|}\mathbf{u}$ ).
3. The *normal vector* of a plane (a vector that is orthogonal to every vector on the plane).
4. The *reduced row-echelon form* (RREF) of a matrix (each row has first non-zero entry a 1, called a *leading 1*, and wherever there is a leading 1 the other entries in the column are all 0, the leading 1s are in a “staircase” pattern, and any all-zero rows are at the bottom).
5. A *consistent* system and an *inconsistent* system of linear equations.

## 2 Things you should be able to do

1. Find the dot product and cross product of vectors.
2. Find the *norm* of a vector.
3. Find the parametric equation for a line.
4. Find the equation of a plane in  $\mathbb{R}^3$ .
5. Find the intersection of a line and a plane in  $\mathbb{R}^3$ .
6. Find the intersection of several planes in  $\mathbb{R}^3$ .
7. Give the the coefficient matrix and the augmented matrix for a system of linear equations.
8. Carry out elementary row operations, and row-reduce a matrix to its RREF.
9. Give all solutions to a system of linear equations. You should also be able to tell when a system has infinitely many solutions, one unique solution or no solutions.
10. Add and subtract matrices.
11. Multiply a matrix by a scalar.
12. Multiply two matrices.

### 3 Results you should know

1. If  $\mathbf{v}$  is a vector in  $\mathbb{R}^n$  then
  - (a)  $\|\mathbf{v}\| \geq 0$ .
  - (b)  $\|\mathbf{v}\| = 0$  if and only if  $\mathbf{v}$  is the zero vector.
  - (c) for a scalar  $k$ ,  $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$ .
2. Vectors  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal if and only if  $\mathbf{u} \bullet \mathbf{v} = 0$ .
3. The zero vector is orthogonal to every vector.
4.  $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .
5. Properties of dot product: for vectors  $\mathbf{u}$  and  $\mathbf{v}$  and  $k$  a scalar
  - (a)  $\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$ ;
  - (b)  $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$ ;
  - (c)  $k(\mathbf{u} \bullet \mathbf{v}) = (k\mathbf{u}) \bullet \mathbf{v}$ .
6. Properties of cross product:
  - (a)  $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$ ;
  - (b)  $\mathbf{u} \bullet (\mathbf{u} \times \mathbf{v}) = 0$  and  $\mathbf{v} \bullet (\mathbf{u} \times \mathbf{v}) = 0$
  - (c)  $\mathbf{u} \times \mathbf{u} = \mathbf{0}$  (i.e. the zero vector).

*R. F. Bailey, 5 February 2012*