UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
Math122–003 Linear Algebra I
Practice questions

These questions are for practice only: they cover topics covered since the last assignment that may appear on the final exam. They do not need to be handed in, but if you want me to go over them with you, you should bring them to my office hours. I will post solutions in a few days.

1. For each of the following matrices, find a basis for, and the dimension of: (i) the row space; (ii) the column space; (iii) the null space.

   (a) $A = \begin{bmatrix} -1 & 1 & 0 & 2 \\ 3 & 1 & 1 & 10 \\ 2 & 4 & -1 & 0 \end{bmatrix}$;

   (b) $B = \begin{bmatrix} -1 & 0 & 2 & 0 \\ 2 & 0 & 1 & 5 \\ 1 & 0 & 0 & 2 \end{bmatrix}$;

   (c) $C = \begin{bmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$.

2. Which of the following linear transformations are one-to-one? *(Hint: use the standard matrix.)*

   (a) $R : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $R(x,y,z) = (2x+y, z-y, x+y+z)$;

   (b) $S : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $S(x,y,z) = (x,x,y-x)$;

   (c) rotation in $\mathbb{R}^2$ counterclockwise by 126.6°;

   (d) dilation in $\mathbb{R}^2$ by 3 followed by an orthogonal projection onto the vector $(-1,3)$.

3. For each of the following matrices, (i) find all the eigenvalues, and then (ii) find bases for the each corresponding eigenspace.

   (a) $A = \begin{bmatrix} -7 & -12 \\ 6 & 10 \end{bmatrix}$;

   (b) $B = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$;

   (c) $C = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$;

   (d) $D = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 4 & -4 & 9 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

R. F. Bailey, 7th December 2009