

## Midterm Test 2

Time: 70 minutes

Instructor:

Dr. Edward Doolittle

Name: \_\_\_\_\_

Student #: \_\_\_\_\_

Section: \_\_\_\_\_

You have 70 minutes to do each of the following questions. The test is worth a total of 60 marks. Non-programmable calculators, logarithm tables, and/or slide rules are permitted; no other aids are permitted. Use the backs of the pages for rough work.

1. Differentiate the following functions.

[3] (a)  $y = (x^2 + 3)(x^2 - 3)^{10}$

[3] (b)  $y = \frac{1}{\sqrt[3]{x} + 1}$

[4] (c)  $y = \left( \frac{x}{x^2 + 1} \right)^2$

2. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot.

[2] (a) Draw a diagram illustrating the situation and labeling the significant dimensions of the problem with letters.

[3] (b) Identify the constraint and the objective for the problem.

[4] (c) Write the objective in terms of a single variable, and find its first and second derivatives.

[6] (d) Find the dimensions of the garden such that the cost of the materials is minimized. (Don't forget to check that you really do have a minimum and not a maximum!)

3. A pharmacist wants to establish an optimal inventory control policy for a new antibiotic. The pharmacist expects to sell 800 packages of this antibiotic at a steady rate during the next year. She plans to place several orders of the same size spaced evenly throughout the year. The ordering cost for each delivery is \$16, and the carrying cost (for refrigeration, rent, and insurance) is \$4 per year per package, based on the average number of packages in stock.

- [5] (a) Let  $x$  be the order quantity and  $r$  the number of orders placed during the year. Find the inventory cost  $C$  (ordering cost plus carrying cost) in terms of  $x$  and  $r$ .
- [5] (b) Find the relationship between  $x$  and  $r$ , and use that constraint to find  $C$  as a function of  $x$  alone. Then find  $C'(x)$  and  $C''(x)$ .
- [5] (c) Determine the minimum inventory cost.

4. A sugar refinery can produce  $x$  tonnes of sugar per week at a weekly cost of

$$C(x) = 0.1x^2 + 5x + 2250$$

dollars.

[5] (a) Find the level of production for which the **average** cost per tonne is minimum.

[5] (b) Show that the average cost equals the marginal cost at that level of production.

- [5] 5. Find the coordinates of all points on the curve  $y = (-x^2 + 4x - 3)^3$  with a horizontal tangent line.

- [5] 6. Find the  $x$ -intercepts of  $y = x^3 - 7x + 6$ . Hint: when  $x = 1$ ,  $y = 0$ .