MATH 103 Problem Set 1 Solutions DRAFT

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1. (a) Putting the equation in the form $y = mx + b$, i.e., $y = 3x + 4$, we see that the slope is $m = 3$ and the y-intercept is $b = 4$.

   (b) Putting the equation in the form $y = mx + b$, i.e., $y = 0x + 5$, we see that the slope is $m = 0$ and the y-intercept is $b = 5$.

   (c) Solving for y we have

   $$y = -\frac{5}{2}x - 1$$

   so the slope is $-5/2$ and the y-intercept is -1.

2. (a) Since the slope is 5 the equation is of the form $y = 5x + b$. Since $(1, 2)$ is on the line we have $2 = 5(1) + b$ which implies $b = -3$. In summary, the required equation is $y = 5x - 3$.

   (b) We can reduce this problem to a problem similar to the previous by finding what the slope of the line must be

   $$\frac{\text{rise}}{\text{run}} = \frac{5-1}{2-1/2} = \frac{4}{3/2} = \frac{8}{3}.$$

   Therefore the line is of the form $y = (8/3)x + b$. Substituting either of the points we know into the equation, say $(2, 5)$ (try it with the other to see what happens), we obtain $5 = (8/3)(2) + b$ which implies $b = -1/3$, so an equation for the line is $y = (8/3)x - (1/3)$. Another correct answer, which doesn't use fractions, would be $8x - 3y = 1$. You should check your solution by checking that the line does indeed pass through the given points.

   (c) The slope of the line $y - x = 4$ is 1 (why?). Any parallel line has the same slope, so the line we are looking for is of the form $y = 1x + b$. Now the question reduces to one similar to part (a); substituting the given point into what we know about the equation we obtain $-5 = 1(3) + b$ which implies $b = -8$. So an equation for the line we are looking for is $y = 1x - 8$, which will usually be written $y = x - 8$.

   (d) Saying that the x-intercept is 2 is the same as saying that the line passes through the point $(2, 0)$. Saying that the y-intercept is -6 is the same as saying that the line passes through $(0, -6)$. Now the problem reduces to something like part (b) above. We find the slope of the line as

   $$m = \frac{\text{rise}}{\text{run}} = \frac{-6-0}{0-2} = \frac{-6}{-2} = 3.$$

   We know that the slope of the line is $m = 3$ and the y-intercept is $b = -6$, so the equation must be $y = 3x - 6$. You should check that that line has all the required properties.

3. (a) Assuming that the daily cost C is a linear function of the daily production x we have $C(x) = mx + b$ for some m and b. The fixed cost is \$2500, so that $y = C(x)$ passes through the point $(0, 2500)$ so $b = 2500$. Furthermore, the line $y = C(x)$ also passes through $(110, 3600)$ so the slope is

   $$m = \frac{3600-2500}{110-0} = \frac{1100}{110} = 10$$

   so the cost function is $C(x) = 10x + 2500$.

(b) The marginal cost is the slope of the line, i.e., 10 dollars per unit produced in a day.

(c) The cost of producing one more unit is the marginal cost, i.e., 10 dollars. The hard way to answer this question is to figure out

$$C(151)-C(150)=(10(151)+2500)-(10(150)+2500)=(1510+2500)-(1500+2500)=4010-4000=10,$$

i.e., the same answer as we obtained by the easy way.

4. (a) It is a good idea to double check that the point is actually on the curve. We do so by checking that $y=x^2$ for the given point (x,y), i.e., $4=2^2$, which is true, so the point is on the curve. The slope of the tangent line at any point is equal to the derivative, which by the power rule is $y'=2x$. So at the point $(2,4)$ where $x=2$ we have $y'=2x=2(2)=4$. Therefore the tangent line has equation $y=4x+b$. Using the fact that the tangent line passes through the given point $(2,4)$ we have $4=4(2)+b$ which implies $b=-4$. Therefore the equation of the tangent line is $y=4x-4$.

   (b) Double check that $(0,0)$ is on the curve: $0=0^2$ so it is. In this case $m=y'=2(0)=0$, and the tangent line has equation $y=0x+b$. Since the tangent line passes through $(0,0)$ we have $0=0(0)+b$ which implies $b=0$. Therefore the equation of the tangent line is $y=0x+0$, i.e., $y=0$.

   (c) Double check that $(-2/5,4/25)$ is on the curve: $4/25=(-2/5)^2$ so it is. Here $m=y'=2x=2(-2/5)=-4/5$ and the tangent line has equation $y=(-4/5)x+b$. Since the curve passes through $(-2/5,4/25)$ we have $4/25=(-4/5)(-2/5)+b$ which implies $b=-4/25$ and the equation of the tangent line is $y=(-4/5)x-4/25$.

5. This question is similar to the previous, only in this case we are not given the point (x,y), only the x value -0.8, so we have to calculate the y value from the function: $y=x^2=(-0.8)^2=0.64$ so the curve and the tangent line pass through the point $(-0.8,0.64)$. Now we can continue as we did above. The slope of the tangent line is $m=y'=2x=2(-0.8)=-1.6$. The tangent line has equation $y=-1.6x+b$. Since it passes through $(-0.8,0.64)$ we have $0.64=-1.6(-0.8)+b$ which implies $b=-0.64$ so the equation of the tangent line is $y=-1.6x-0.64$.

6. The slope of $y=x^2$ is $y'=2x$. Since we are given that the slope is -6 we have the equation $-6=y'=2x$, i.e., $-6=2x$. That implies that $x=-3$. To find the point on the curve we need to find the corresponding y coordinate: $y=x^2=(-3)^2=9$. So the required point is $(-3,9)$.

7. Two lines are parallel if and only if they have the same slope. The slope of the given line is $m=-1/9$ (we find that by solving for y: $y=-1/9x+7/9$. Now we can solve the problem in the same manner as we solved the previous problem. We have $y'=2x$ so $-1/9=2x$ so $x=-1/18$, and the point on the curve is $(-1/18,1/324)$.

8. This question is similar to question 4, except that we can stop once we have the slopes. It is a good idea to double check that the given points are on the curve, but I'll leave that up to you.

   (a) By the power rule, the derivative of the function is $y'=3x^2$ so the slope of the curve is $y'=3(3)^2=27$.

   (b) The slope of the curve is $y'=3(-1)^2=3$.

   (c) The slope of the curve is $y'=3(1/2)^2=3/4$.

9. The derivative is $y'=3x^2$ so the slope of the curve (which is the same as the slope of the tangent line) is $m=y'=3x^2=3(-2)^2=12$. The tangent line has an equation of the form $y=12x+b$. We still need a point on the line, which will be the point on the curve where $x=-2$. At that point we have $y=x^3=(-2)^3=-8$ so the tangent line passes through $(-2,-8)$. Therefore we must have $-8=12(-2)+b$ which implies $b=16$ and the line is $y=12x+16$.

10. This question is similar to question 7. The slope of the tangent line must be the same as the slope of the given line, which is 12. From $y'=3x^2$ we have $12=3x^2$, $x^2-4=0$, $(x+2)(x-2)=0$ giving two possibilities for x. The possibility $x=-2$ was covered in the previous problem; the equation of the tangent line in this case is $y=12x+16$. For $x=2$ we have $y=x^3=8$ so the tangent line passes through $(2,8)$ and is of the form $y=12x+b$. It follows that $8=12(2)+b$ so $b=-16$, and the second tangent line satisfying the given conditions is $y=12x-16$. Note that there are two answers to this question!