MATH 103 Problem Set 3 Solutions DRAFT

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1. (a) Write $f(x) = 3(3x^4 + x + 1)^{1/2}$. Then by the constant multiple and general power rules,

$$f'(x) = 3 \cdot \frac{1}{2}(3x^4 + x + 1)^{-1/2} \cdot \frac{d}{dx}(3x^4 + x + 1) = \frac{3}{2}(3x^4 + x + 1) \cdot (12x^3 + 1).$$

   (b) Write $g(x) = (2x+5)^{-1}$. Then by the general power rule,

$$f'(x) = -1 \cdot (2x+5)^{-2} \cdot \frac{d}{dx}(2x+5) = -1 \cdot (2x+5)^{-2} \cdot 2.$$

   (c) Write $h(x) = (x + x^{-1})^{-2}$. Then

$$h'(x) = -2(x + x^{-1})^{-3} \cdot \frac{d}{dx}(x + x^{-1}) = -2(x + x^{-1})^{-3} \cdot (1 + (-1)x^{-2}),$$

   where the final step used the ordinary power rule.

2. We can find $h(3)$ without differentiating. We have

$$h(3) = 5f(3) - 4g(3) = 5(6) - 4(-2) = 38.$$

   To find h' we need to differentiate the formula for h. By the sum and constant multiple rules,

$$h'(x) = \frac{d}{dx}(5f(x) - 4g(x)) = \frac{d}{dx}5f(x) - \frac{d}{dx}4g(x) = 5\frac{d}{dx}f(x) - 4\frac{d}{dx}g(x) = 5f'(x) - 4g'(x).$$

   Therefore we have $h'(3) = 5f'(3) - 4g'(3) = 5(1) - 4(7) = -23$.

3. We have $f(1) = 2\sqrt{g(1)} = 2\sqrt{5}$. Writing $f(x) = 2(g(x))^{1/2}$ and differentiating, we have

$$f'(x) = 2 \cdot \frac{1}{2}(g(x))^{-1/2} \cdot g'(x)$$

   so $f'(1) = (g(1))^{-1/2} \cdot g'(1) = 5^{-1/2} \cdot -3$.

4. By the general power rule,

$$y' = 3(x+1)^2 \cdot \frac{d}{dx}(x+1) = 3(x+1)^2.$$

   The points where the curve has slope 75 are the points where $75 = y' = 3(x+1)^2$. Solving for x we have $75 = 3(x+1)^2$ which implies $25 = (x+1)^2$ which implies $x + 1 = \pm 5$, $x = -1 \pm 5 = -6 or 4$. The actual points on the curve can be found by substituting those values back into the function to obtain $(x, y) = (-6, -125)$ or $(4, 125)$.

5. (a) First we find the first derivative:

$$f'(x) = 2(1-3x^2)^1 \cdot \frac{d}{dx}(1-3x^2) + 4x^3 = 2(1-3x^2)(-6x) + 4x^3.$$

Since we don't yet know how to differentiate a product, we have to simplify the above expression before we can differentiate again. Applying the distributive law,

$$f'(x) = -12x + 36x^3 + 4x^3 = -12x + 40x^3.$$

Differentiating a second time gives $f''(x) = -12 + 120x^2$.

(b) We first find the first derivative:

$$\frac{d}{dx}(x^5 + 3x^2) = 5x^4 + 6x.$$

Next we find the second derivative:

$$\frac{d^2}{dx^2}(x^5 + 3x^2) = \frac{d}{dx}\left(\frac{d}{dx}(x^5 + 3x^2)\right) = \frac{d}{dx}(5x^4 + 6x) = 20x^3 + 6.$$

Finally we evaluate the second derivative at the value $x = 2$ to obtain $20(2)^3 + 6 = 20 \cdot 8 + 6 = 166$.

(c) Write $g(x) = (x-3)^{-1}$. Then

$$g'(x) = -(x-3)^{-2}\frac{d}{dx}(x-3) = -(x-3)^{-2}$$

$$g''(x) = -1 \cdot -2(x-3)^{-3} \cdot \frac{d}{dx}(x-3) = 2(x-3)^{-3}$$

so $g''(1) = 2(1-3)^{-3} = 2(-2)^{-3} = 2 \cdot -1/8 = -1/4$.

(d) Here a, b, and c are constants so we can write

$$h'(x) = \frac{d}{dx}(ax^2 + bx + c) = \frac{d}{dx}ax^2 + \frac{d}{dx}bx + \frac{d}{dx}c = a\frac{d}{dx}x^2 + b\frac{d}{dx}x + 0 = 2ax + b$$

$$h''(x) = \frac{d}{dx}(2ax + b) = 2a.$$

6. For this type of question we use the estimate

$$C(x+h) \approx C(x) + hC'(x),$$

here with $x = 50$ and $h = 5$, giving us $C(55) \approx C(50) + 5C'(50) = 5000 + 5 \cdot 45 = 5225$.

7. Again, we use the estimate

$$f(x+h) \approx f(x) + hf'(x)$$

with $x = 25$ and with varying values of h. In particular, we have

$$f(27) \approx f(25) + 2f'(25) = 10 + 2(-2) = 6$$
$$f(26) \approx f(25) + 1f'(25) = 10 + 1(-2) = 8$$
$$f(25.25) \approx f(25) + 0.25f'(25) = 10 + 0.25(-2) = 9.5$$
$$f(24) \approx f(25) - 1f'(25) = 10 - 1(-2) = 12$$
$$f(23.5) \approx f(25) - 1.5f'(25) = 10 - 1.5(-2) = 13.$$

Note that in the last two cases h takes on negative values, namely -1 and -1.5.

8. (a) $S(1) = 120.560$ and $S'(1) = 1.500$.

(b) The first part means that $S(3) = 80.000$ but the second part is rather more difficult to interpret because the information is given in terms of days rather than months. One way to answer the question is to write the information in terms of change per month: sales are falling by \$200 per day means that they are falling at a rate of $30 \times \$200 = \$6000$ per month, i.e., $S'(3) = -6$.