

MATH 103 200710 Problem Set 5

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Thursday, March 8, 2007

The following problems may appear on Midterm Test 2 on Thursday, March 29, 2007.

1. Find the maximum value of each of the following functions, and state for what value of the argument the maximum actually occurs.

(a) $f(t) = 24t - t^2$

(b) $g(x) = 10 + 4x - x^2$

(c) $h(t) = -t^3 + 6t^2 + 40, t \geq 0$

2. Find the maximum value of Q in each of the following, subject to the given constraint:

(a) $Q = xy; x + y = 2$

(b) $Q = xy^2; x + y = 4, x, y \geq 0$

(c) $Q = -x^2 - y^2; x + y = 6$

3. Eight hundred dollars are available to fence in a rectangular garden. One side of the garden faces a road, and the cost of the fencing for that side is \$8 per metre. The fencing for the other three sides is \$6 per metre. We are interested in maximizing the area of the garden.

(a) Determine the objective and constraint equations.

(b) Express the quantity to be maximized as a function of the length of the side of the garden facing the road.

(c) Find the dimensions of the garden which maximize the area.

4. A rectangular garden of area 300 square feet is to be surrounded on three sides by a cement wall costing \$30 per metre, and one side by a fence costing \$15 per metre. Find the dimensions of the garden for which the cost of materials is minimized.

5. A bookstore is attempting to determine the economic order quantity for a popular book. The store sells 4000 copies of the book a year. The store figures that it costs \$40 to process each new order for books. The carrying cost (due to interest on the money borrowed to purchase the books and insurance) is about \$2 per book on the average number of books in stock.

(a) How many times a year should a batch of books be ordered? What is the economic order quantity?

(b) Suppose the book is featured on Oprah in December, and demand for the following year quadruples. Now what is the economic order quantity?

6. The demand function for a certain product is $p = x^2/12 - 10x + 300$ in dollars for x units produced, where x is restricted to the interval $0 \leq x \leq 60$. Find the level of production and corresponding price p such that the revenue is maximized.

7. A dance club offers memberships at a cost of \$100, provided that a minimum of 500 people join. For each member in excess of 500, the membership fee will be reduced by \$1. How many memberships should the club sell to maximize its revenue?

8. A nightclub charges \$8 admission, and has an average of 1200 people going through the doors per night. One week they reduced the admission charge to \$7 and noticed that 1700 people went through the doors that evening.

(a) Assuming that the demand function $p(x)$ is linear, find it based on the above data.

- (b) What should the nightclub charge to maximize its revenue?
 - (c) Suppose there is are fixed costs of \$2000 per night, and marginal costs (security, insurance) of \$0.40 per person admitted. What should the nightclub charge to maximize its profit?
 - (d) Suppose fire regulations cap the number of persons allowed in the building to 2400. Now what should the nightclub charge to maximize its profit?
9. A certain airline requires that rectangular packages carried on an airplane be such that the sum of the three dimensions is at most 120 centimetres. What are the dimensions of a square-bottomed rectangular package of maximum volume meeting the airline's requirements?

Please do the following problems from the textbook. They may appear on Midterm Test 2.

2.5 C-level: 1–9, 11, 14–15; B-level: 10, 12–13, 16–26; A-level: 27–31.

2.6 C-level: 1–8; B-level: 9–22, 24, 26; A-level: 25, 27–28.

2.7 C-level: 1–9, 11–16; B-level: 10, 17–19; A-level: 20–22.