MATH 103 Problem Set 9 Solutions DRAFT

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1. (a) By the sum, constant multiple, and power rules for integrals we have

$$\int \left(2x^2 - \frac{1}{3x}\right)\,dx = 2\int x^2\,dx - \frac{1}{3}\int \frac{1}{x}\,dx = \frac{2}{3}x^3 - \frac{1}{3}\ln x + C.$$

   (b) By the rule for $\int e^{kx}\,dx$ applied with $k = -1$ we have

$$\int e^{-x}\,dx = \frac{e^{-x}}{-1} + C = -e^{-x} + C.$$

   (c) Performing a little algebra to simplify the integrand and then applying familiar rules for integration we have

$$\int -2(e^{2x}+1)\,dx = \int (-2e^{2x} - 2)\,dx = -2\int e^{2x}\,dx - 2\int 1\,dx = -2\frac{e^{2x}}{2} - 2x + C = -e^{2x} - 2x + C.$$

   (d) Performing a little algebra to simplify the integrand and then applying familiar rules for integration we have

$$\int x\sqrt{x}\,dx = \int x^1 \cdot x^{1/2}\,dx = \int x^{3/2}\,dx = \frac{x^{5/2}}{5/2} + C = \frac{2}{5}x^{5/2} + C.$$

2. For each of these problems, we first evaluate the corresponding indefinite integral (antiderivative) and then evaluate the definite integral by the Fundamental Theorem of Calculus.

   (a) We have

$$\int (4x^3 - 1)\,dx = x^4 - x(+C) \implies \int_0^1 (4x^3 - 1)\,dx = (x^4 - x)\Big|_0^1 = (1^4 - 1) - (0^4 - 0) = 0.$$

   (b) We have

$$\int 4e^{-3x}\,dx = -\frac{4}{3}e^{-3x}(+C)$$

   which implies

$$\int_1^4 4e^{-3x}\,dx = \left(-\frac{4}{3}e^{-3(4)}\right) - \left(-\frac{4}{3}e^{-3(1)}\right) = \frac{4}{3}(e^{-3} - e^{-12}).$$

   (c) We have $\int x^{-1}\,dx = \ln x(+C)$ so

$$\int_3^6 x^{-1}\,dx = \ln 6 - \ln 3.$$

(d) We have

$$\int \sqrt{e^x}\,dx = \int e^{x1/2}\,dx = \int e^{x/2}\,dx = \frac{e^{x/2}}{1/2} + C = 2e^{x/2} + C.$$

By the Fundamental Theorem of Calculus the definite integral is

$$\int_0^1 \sqrt{e^x}\,dx = 2e^{x/2}\Big|_0^1 = 2e^{1/2} - 2e^{0/2} = 2e^{1/2} - 1.$$

3. (a)
   (b)
   (c)
   (d)

4. (a) Let $u = 2x - 1$. Then $du/dx = 2$ so $du/2 = dx$ and we can write

$$\int (2x-1)^7\,dx = \int u^7\,\frac{du}{2} = \frac{1}{2}\frac{u^8}{8} + C = \frac{1}{16}(2x-1)^8 + C.$$

You should check by differentiating.

(b) In this case, let $u = -x^2$. Then $du = -2x\,dx$ so $-du/2 = x\,dx$ and we have

$$\int xe^{-x^2}\,dx = \int e^{-x^2}\,x\,dx = \int e^u \cdot -\frac{du}{2} = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-x^2} + C.$$

You should check by differentiating.

(c) This problem was done in the sample final exam.

(d) In this case, it's not completely clear what to use as u, so we guess $u = \sqrt{x}$. Then $du = x^{-1/2}/2\,dx$ which implies $2du = dx/\sqrt{x}$. We can carry that substition out to obtain

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int e^u\,2du = 2e^u + C = 2e^{\sqrt{x}} + C.$$

You should check by differentiating.

5. (a)
   (b)
   (c)
   (d)

6. (a)
   (b)
   (c)
   (d) You should make a quick sketch of the two curves on the same graph. The bounded region between the two curves is limited by the points where the curves intersect, which we find by setting $y = x^2 + 4 = 4x + 1$. Solving the equation in x we have

$$x^2 + 4 = 4x + 1 \implies x^2 - 4x + 3 = 0 \implies (x-3)(x-1) = 0 \implies x = 1, 3.$$

Between $x = 1$ and $x = 3$ the higher of the two curves is $y = 4x + 1$ and the lower is $y = x^2 + 4$. Subtracting the higher from the lower and integrating between the intersection points gives

$$A = \int_1^3 \big((4x+1) - (x^2+4)\big)\,dx = \int_1^3 (-x^2 + 4x - 3)\,dx = -\frac{x^3}{3} + 2x^2 - 3x\Big|_1^3.$$

Evaluating, we have

$$A = (-\frac{27}{3} + 2(9) - 9) - (-\frac{1}{3} + 2(1) - 3) = -9 + 18 - 9 + \frac{1}{3} - 2 + 3 = \frac{4}{3}.$$

7.