

MATH 103 Quiz 1 Solutions DRAFT

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1.
 - (a) Since $C(x)$ is a linear function of the production level we have $C(x) = mx + b$ for some as yet unknown parameters m and b . We have $C(0) = 2000$ and $C(50) = 2500$ so $b = C(0) = 2000$ and $m = (2500 - 2000)/(50 - 0) = 500/50 = 10$. In summary, $C(x) = 10x + 2000$.
 - (b) By the above formula $C(100) = 10(100) + 2000 = 3000$.
 - (c) We have $C'(100) =$ the slope of the line $m = 10$.
 - (d) The marginal cost is $C(101) - C(100)$ which we could calculate, but we also know a shortcut: the marginal cost at production level 100 is $C'(100) = 10$ dollars per additional unit produced.
2. The given line can be written in the form $y = -4x + (5/2)$ so it has slope -4 . Now we need to find an x value for which the slope of the curve $y = x^4$ is -4 . We know that the slope of the curve is given by the formula $y' = 4x^3$, so we need to solve the equation $-4 = y' = 4x^3$ which implies $x^3 = -1$, i.e., $x = -1$. Now, we know the slope of the tangent line and the x value of a point on the line, so to find the equation of the tangent line we only need to find the y value corresponding to the x value we have. Substituting into the equation of the curve, $y = (-1)^4 = 1$ so $(-1, 1)$ is the point on the curve that corresponds to the point on the tangent line. We can now find the equation of the tangent line. The quickest way is to use the "point-slope" form of the equation: $y - y_0 = m(x - x_0)$. Filling in what we know, $m = -4$, $(x_0, y_0) = (-1, 1)$ we have $y - 1 = -4(x - (-1))$. That answer is fine, but we can simplify somewhat if we wish to obtain $y = -4x - 3$.