MATH 103 Quiz 2 Solutions DRAFT

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1. Since $36/x$ grows without bound as x approaches 0, $R(x)$ has a vertical asymptote at $x = 0$. Since $36/x$ is negligible as x approaches infinity, $R(x) \approx 4x + 1$ as x approaches infinity, so $R(x)$ has the slant asymptote $y = 4x + 1$. (See the blue lines in Figure 1.)

   To investigate the local minima, we differentiate $R(x)$ to obtain $R'(x) = -36/x^2 + 4$ and $R''(x) = 72/x^3$. When $x = 3$ we have $R'(3) = -36/3^2 + 4 = -36/9 + 4 = -4 + 4 = 0$ so there is a stationary point at $x = 3$; furthermore, $R''(3) = 72/3^3 = 72/27 > 0$ so the stationary point is a minimum. To find it on the graph, we find $y = R(3) = 36/3 + 4(3) + 1 = 25$, so we plot the point $(3, 25)$ and mark a little horizontal line segment around the point to emphasize that it is a stationary point.

   Filling out the graph from the frame provided by the above information, we have Figure 1.

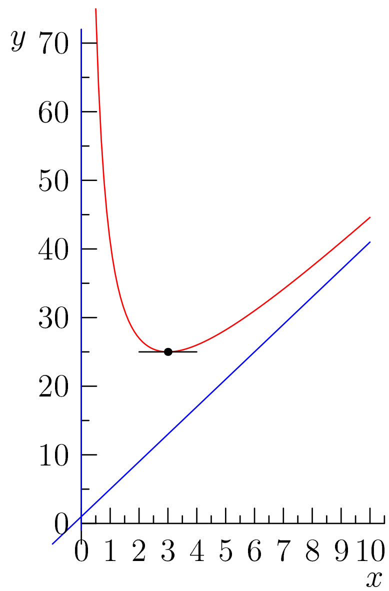

Figure 1: Graph of $y = 36/x + 4x + 1, x > 0$

2. To find the stationary points of the function we find the roots of $f'(x) = -6x^2 - 6x = 0$. Factoring, we have $-6x(x + 1) = 0$, with roots $x = -1$ and $x = 0$. Those are our stationary points.

   To test the nature of the stationary points we differentiate again to obtain $f''(x) = -12x - 6$ and evaluate it at the stationary points. We have $f''(-1) = 6$, so there is a local minimum at $x = -1$. We have $f''(0) = -6$ so there is a local maximum at $x = 0$.

   To find the y values of the stationary points, we evaluate $y = f(-1) = -2(-1)^3 - 3(-1)^2 - 3 = 2 - 3 - 3 = -4$, so in summary, there is a local minimum at $(-1, -4)$. Similarly, $f(0) = -3$ so there is a local maximum at $(0, -3)$. Putting all that information on the graph and connecting the dots with a smooth line we have something like Figure 2.

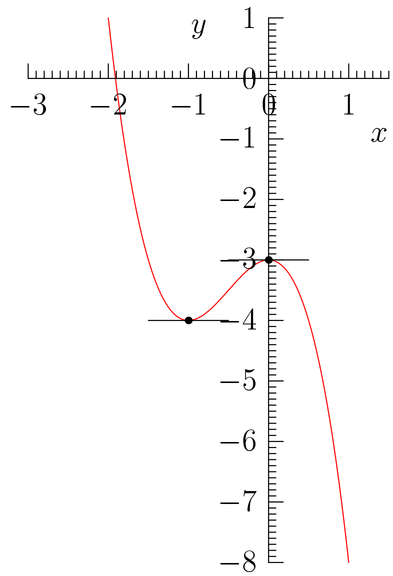

Figure 2: Graph of $y = -2x^3 - 3x^2 - 3$