MATH 103 Quiz 3 Solutions DRAFT

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1. By the quotient rule, the derivative is

$$y' = \frac{(x^3-2)^4 \frac{d}{dx} x^3 - x^3 \frac{d}{dx}(x^3-2)^4}{((x^3-2)^4)^2}.$$

Applying the power rule to differentiate x^3 and the chain rule to differentiate $(x^3-2)^4$, we have

$$y' = \frac{(x^3-2)^4 \cdot 3x^2 - x^3 \cdot 4(x^3-2)^3 \cdot \frac{d}{dx}(x^3-1)}{((x^3-2)^4)^2} = \frac{(x^3-2)^4 \cdot 3x^2 - x^3 \cdot 4(x^3-2)^3 \cdot 3x^2)}{((x^3-2)^4)^2}.$$

There is no need to simplify any further, although for some purposes writing

$$y' = \frac{3x^2\left((x^3-2)-4x^3\right)}{(x^3-2)^5} = \frac{3x^2(-3x^3-2)}{(x^3-2)^5}$$

might be helpful.

2. It is best to write the function in power notation before doing anything else: $y = (x^2+8x+17)^{1/2}$. To find stationary points, we differentiate by the chain rule:

$$y' = \frac{1}{2}(x^2+8x+17)^{-1/2}\frac{d}{dx}(x^2+8x+17) = \frac{1}{2}(x^2+8x+17)^{-1/2}(2x+8).$$

Simplifying, we can write

$$y' = \frac{x+4}{\sqrt{x^2+8x+17}}.$$

Stationary points are found by solving the equation $y' = 0$:

$$\frac{x+4}{\sqrt{x^2+8x+17}} = 0 \implies x+4=0 \implies x=-4.$$

We have now completed the question as posed, although it is a good exercise to continue and answer some questions which may typically be asked at this juncture: where is the stationary point located on the (x,y)-plane, and is it a minimum or a maximum? The answer to the first question is $(-4,1)$, and the answer to the second question is a minimum, by the first derivative test. How are those answers obtained, and why is the first derivative test superior to the second derivative test in this situation?