MATH 103 Quiz 3 Solutions DRAFT

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1. The formula for the value of the bank account at time t in years is $A = 1000e^{0.06t}$.

   (a) The balance reaches \$2500 when $A = 2500$, in which case we have the equation

   $$2500 = 1000e^{0.06t} \implies 2.5 = e^{0.06t} \implies \ln(2.5) = 0.06t \implies t = \frac{\ln(2.5)}{0.06} = 15.27.$$

   The balance will be \$2500 just after 15 years, 3 months have passed.

   (b) The best way to solve this problem is from the differential equation. We have $A(t) = 1000e^{0.06t}$, so $A'(t) = 0.06 \times 1000e^{0.06t} = 0.06A(t)$. When the balance is \$2500, we have $A'(t) = 0.06A(t) = 0.06 \times 2500 = 150$ dollars per year.

   A more straightforward, but involved, way to solve the problem is to use the formula $A'(t) = 0.06 \times 1000e^{0.06t}$ and the time $t = 15.27$ found in part (a). Then $A'(15.27) = 0.06 \times 1000e^{0.06\times 15.27} = 149.99$ dollars per year. There is a slight error due to rounding t, but that is OK because these results are approximate anyway.

2. Simplifying, we have

   $$y = \ln[(x^2+3)(x^3+1)^{-1}] = \ln(x^2+3) - \ln(x^3+1)$$

   by the rules for simplifying logarithms. Differentiating by the chain rule we have

   $$y' = \frac{1}{x^2+3}\frac{d}{dx}(x^2+3) - \frac{1}{x^3+1}\frac{d}{dx}(x^3+1) = \frac{2x}{x^2+3} - \frac{3x^2}{x^3+1}.$$