UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS

Mathematics 103–001, 002, S01, S02
Final Examination, Fall 2006

Time: 3 hours
Instructors:
D. Farenick (001)
C. Guo (002)
S. Lisawadi (S01, S02)

Name: ______________________________

Student No.: ________________________

Section: ____________________________

Show all your work and explain all your answers. Use the back of each page if sufficient space is not available. Use scrap paper for rough work, and do not hand it in. No graphing or programmable calculators are allowed.

1. (6 marks) Use the definition of the derivative to compute $f'(1)$ when

$$f(x) = \sqrt{x+3}.$$

2. (5 marks) Let

$$f(x) = \begin{cases} e^{x^2-1} & \text{if } x \neq -2 \\ e^2 & \text{if } x = -2 \end{cases}$$

Is $f(x)$ continuous at $x = -2$? Why or why not?

3. (8 marks) Evaluate the following limits.

(a) $\lim_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 - 4}$

(b) $\lim_{x \to \infty} \dfrac{x^2 - 4x + 3}{5x^2 + 6}$

4. (12 marks) Find the indicated derivatives.

(a) $f(x) = (x^2 + 1)^{3/2}\,(2x - 3); \quad f'(x) = ?$

(b) $g(t) = \dfrac{3t + 2}{4t^2 + 1}; \quad g'(t) = ?$

(c) $y = 2e^x + e^3 - \ln(1 + x^2)$; $\quad \dfrac{dy}{dx} = ?$

(d) $f(x) = e^{x^2+1}$; $\quad f''(0) = ?$

5. (6 marks) Let $f(x) = \sqrt{x^2 + 8}$. Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(1, 3)$.

6. (5 marks) Find the critical points of $f(x) = (1 - x)e^{2x}$.

7. (12 marks) Consider the graph of $f(x) = x^3 - 3x^2 + 4$. Identify all relative and absolute extreme points and inflection points. Use your result to sketch a graph of $y = f(x)$.

8. (8 marks) A rectangular garden of area 120 square feet is to be surrounded on three sides by a brick wall costing \$15 per foot and on one side by a fence costing \$10 per foot. Find the dimensions of the garden such that the cost of materials is minimized.

9. (8 marks) A tour operator knows from experience that 50 seats on a particular tour can be sold at \$200 per seat and that each \$2 price reduction will yield one additional seat sale. Operating the tour has a fixed cost of \$5000 and an additional cost of \$36 per person. Assuming that price is a linear function, determine the price that the tour operator should charge per seat to maximize the profit.

10. (6 marks) The value of a computer t years after purchase is $v(t) = 2000e^{-0.35t}$ dollars. When will the value of the computer be decreased to half of its purchase value?

11. (8 marks) Sketch the region bounded by the curves $y = 3 - x^2$ and $y = 2x - 5$, and then find the area of the region.

12. (16 marks) Evaluate the following integrals.

(a) $\int (\frac{2}{x} + 8x^3 - 6e^x)\ dx.$

(b) $\int_0^{\ln 3} 9e^{-3x}\ dx.$

(c) $\int \frac{x^3}{1+x^4}\ dx.$

(d) $\int e^x \sqrt{1+e^x}\ dx.$