

MATH111-002 200530 Practice Midterm 1

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The following questions will give you an idea of what to expect on Midterm 1, scheduled for Thursday, September 29, 2005. The test should take you about 50 minutes; you should be able to earn about one mark per minute, so the test is out of 50. In addition to this practice midterm, please try the textbook practice problems listed at the end of this document.

About 10% of this test (5 marks out of 50) is very difficult 'A level' work. Another 10–20% (5–10 marks out of 50) is fairly difficult 'B level' work. The rest should be straightforward, although to do the test in the allotted time you will probably need to do a lot of practice problems to refresh your memory.

1. (10 marks) Find the following derivatives:

- (a) $h'(x)$ where $h(x) = e^x(\sin x - \cos x)$
- (b) $(f^{-1})'(1)$ where $f(x) = e^{\sin x}$, $-\pi/2 < x < \pi/2$. (Hint: what is $f(0)$?)
- (c) $g''(x)$ where $g(x) = e^{e^x}$.
- (d) y' where $e^{xy} = x^2 - y$ (express your answer as a function of x and y).
- (e) $\frac{d}{dt} \int_{-t}^t e^{x^2} dx$.

2. (8 marks) Find the following integrals:

- (a) $\int (1-u)^8 du$.
- (b) $\int_0^5 e^{-3x} dx$.
- (c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.
- (d) $\int_0^\pi \sin x \cos(\cos x) dx$.

3. (4 marks) Show that $(0, 1)$ is a point on the graph of $y = e^{2x} \cos \pi x$, and find an equation of line tangent to the graph at that point.

4. (5 marks) Find an equation of a tangent line to the graph of $y = \frac{x-1}{x+1}$ which is perpendicular to the line $2x + y = 1$.

5. (4 marks) Find the absolute maximum value of the function $p(x) = x - e^x$.

6. (8 marks) Let $q(x) = x^3 - x$. Sketch the graphs of $y = q(x)$ showing intercepts, local maxima, minima and points of inflection. You should also find regions where the function is increasing, decreasing, concave up, and concave down.

7. (6 marks) Let $r(x) = 3x$. Find the area of the finite region bounded by the graphs of $y = q(x)$ (from the previous question) and $y = r(x)$. Hint: you need to find the points of intersection where $q(x) = r(x)$.

8. (5 marks) Evaluate $\lim_{x \rightarrow \pi} \frac{e^{\sin x} - 1}{x - \pi}$.

You should do the following extra practice problems in Stewart, 5e, in preparation for the test, as well as the problems from 7.1 and 7.2 that I recommended in Problem Set 1. You do not need to hand these problems in!

The sections 3.R, 4.R, and 5.R can be found in Stewart 5e on pages 213, 308, and 368 respectively.

C-level Problems you should be able to do if you want to pass the course.

3.R: 13–40, 45–47, 51, 57–58

4.R: 1–6, 17–25 (don't do too many, they are long), 38–47

5.R: 9–24, 33–34

6.1: 1–19

B-level Problems you should be able to do if you want a grade in the 70s.

3.R: 41–44, 48, 52–55, 59–67, 84–86

4.R: 7–12, 26–28, 33–37

5.R: 7–8, 25–28, 35–40 (Property 8 is on page 335)

6.1: 20–30, 41, 44, 50, 52

A-level Problems you should be able to do if you want a grade in the 80s or 90s.

3.R: 87–90

4.R: 62–63

5.R: 41–42, 49, 51–56

6.1: 43, 45–48