MATH 111 Sample Final Examination Solutions DRAFT

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1. (a) By the product rule, the rules for differentiating e^x and $\sin^{-1}$, and the chain rule,

$$y' = e^x \sin^{-1}(3x) + e^x \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

(b) By the quotient rule and the rule for differentiating $\tan^{-1}$,

$$y' = \frac{0 - 1 \cdot 1/(1+x^2)}{(\tan^{-1} x)^2}.$$

Alternatively you can write $y = (\tan^{-1} x)^{-1}$. Note that the -1 inside the bracket means something completely different from the -1 outside the bracket: inside the bracket it indicates an inverse function, outside the bracket it is an exponent of -1. Then by the chain rule,

$$y' = -1(\tan^{-1} x)^{-2} \cdot \frac{d}{dx} \tan^{-1} x = -(\tan^{-1} x)^{-2} \frac{1}{1+x^2}$$

which is the same as the first answer.

(c) Logarithmic differentiation is helpful here. Taking the logarithm of both sides of the definition of y,

$$\ln y = \ln(x+\sin x)^x = x \ln(x + \sin x)$$

using the one of the laws of logarithms. Differentiating both sides,

$$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln(x + \sin x);$$

differentiating the left hand side implicitly and differentiating the right hand side using the product rule and chain rules,

$$\frac{y'}{y} = \ln(x+\sin x) + x \cdot \frac{1}{x+\sin x} \cdot \frac{d}{dx}(x+\sin x) = \ln(x+\sin x) + x \cdot \frac{1}{x+\sin x} \cdot (1+\cos x).$$

Solving for y',

$$y' = y\left(\ln(x+\sin x) + x \cdot \frac{1}{x+\sin x} \cdot (1+\cos x)\right) = (x+\sin x)^x \left(\ln(x+\sin x) + x \cdot \frac{1}{x+\sin x} \cdot (1+\cos x)\right).$$

2. (a) **Solution 1.** The limit is of the form $\infty - \infty$. Try to change it into a limit of the form $0/0$ by finding a common denominator and adding fractions:

$$L = \lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \lim_{x\to 0} \left(\frac{e^x - 1}{x(e^x-1)} - \frac{x}{x(e^x-1)}\right) = \lim_{x\to 0} \frac{e^x - 1 - x}{xe^x - x}.$$

The latter limit above is of the form $0/0$ so we can apply L'Hôpital's rule:

$$L = \lim_{x\to 0} \frac{e^x - 1}{e^x + xe^x - 1}.$$

The limit is still of the form $0/0$ so we can apply L'Hôpital's rule again to obtain

$$L = \lim_{x\to 0} \frac{e^x}{e^x + e^x + xe^x} = \frac{1}{2}.$$

Solution 2 The Maclaurin series for e^x is $e^x = 1 + x + x^2/2 + x^3/6 + \cdots$, so we have

$$L = \lim_{x\to 0} \frac{e^x - 1 - x}{xe^x - x} = \lim_{x\to 0} \frac{1 + x + x^2/2 + x^3/6 + \cdots - 1 - x}{x + x^2 + x^3/2 + x^4/6 + \cdots - x} = \lim_{x\to 0} \frac{x^2/2 + x^3/6 + \cdots}{x^2 + x^3/2 + \cdots} = \frac{1}{2}.$$

(b) The limit is of the form ∞/∞ so we can apply L'Hôpital's rule to obtain

$$L = \lim_{x\to 0^+} \frac{1 - \ln x}{e^{1/x}} = \lim_{x\to 0^+} \frac{-1/x}{e^{1/x}(-1/x^2)} = \lim_{x\to 0^+} \frac{x}{e^{1/x}}.$$

The limit is now of the form $0/\infty$ so $L = 0$.

3. (a) **Solution 1.** The function is defined for all $x \geq 0$, so in order to show that it is one-to-one we just need to show that it is monotone (strictly increasing or decreasing) which we can do using the first derivative.

$$f'(x) = e^{x\tan^{-1}x} \cdot \frac{d}{dx} x\tan^{-1}x = e^{x\tan^{-1}x} \cdot \left(\tan^{-1}x + \frac{x}{1+x^2}\right).$$

The first factor of the above is an exponential, so it is always positive. The second factor is always positive because all of $\tan^{-1}x$, x, and $1+x^2$ are positive for positive values of x. It follows that the function is monotone. (It doesn't matter that the derivative of the function is 0 at the endpoint of the interval $x = 0$. Try to use the mean value theorem to figure out why.)

Solution 2. The functions $g(x) = x$ and $h(x) = \tan^{-1}(x)$ are both increasing and positive on the interval $[0,\infty)$, so their product is also (if $0 \leq a < b$ then $0 \leq \tan^{-1}(a) < \tan^{-1}(b)$, and multiplying the two inequalities together we obtain $a\tan^{-1}(a) < b\tan^{-1}(b)$). Since the functions e^x and $x\tan^{-1}(x)$ are increasing, it follows that their composition is also increasing (why?).

(b) Since the function is one-to-one it has an inverse f^{-1}. By the formula for differentiating the inverse of a function,

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

whenever it is defined. We worked out $f'(x)$ above. We need to figure out $f^{-1}(1)$ which we often do just by guessing: $f(0) = e^{0\cdot\tan^{-1}0} = e^0 = 1$ so $f^{-1}(1) = 0$. Substituting that into f' we have $f'(f^{-1}(1)) = f'(0) = 0$. Therefore $(f^{-1})'(1)$ does not exist. (To be precise, the derivative of f^{-1} at 1 would not exist no matter what the value of $f'(0)$ because the function f^{-1} is not defined on both sides of 1. However, even the one-sided derivative of f^{-1} from above at 1 does not exist. To be even more precise, the one-sided derivative is $+\infty$, but to really show that properly you should probably use the definition of a one-sided derivative.)

4. Usually when we revolve a function of the form $y = f(x)$ about the y-axis we want to use the shell method to calculate the volume of the solid of revolution. The volume of a cylindrical shell is $2\pi xy\,dx$ where x is its radius, y its height, and dx its width. Here $y = \sin x$ and the bounds of integration are $\pi/4$ and π, so the volume is

$$V = \int_{\pi/4}^{\pi} 2\pi x \sin x\, dx.$$

In order to evaluate the corresponding indefinite integral we should integrate by parts with $u = x$, $dv = \sin x\,dx$, $du = dx$, $v = -\cos x$:

$$\int x\sin x\,dx = -x\cos x + \int \cos x\,dx = \sin x - x\cos x.$$

(Check by differentiating.) Substituting that into the expression for V,

$$V = 2\pi\ (\sin x - x\cos x)\Big|_{\pi/4}^{\pi} = 2\pi\left((\sin\pi - \pi\cos\pi) - \left(\sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}\right)\right) = 2\pi\left(\pi - \frac{1}{\sqrt{2}} + \frac{\pi}{4}\frac{1}{\sqrt{2}}\right).$$

(Numerically, I get $V \approx 26.6100$.)

5. (a) On the interval $[1,4]$ the integrand is defined and continuous because it is a product of continuous functions, so the integral is proper. First evaluate the indefinite integral by parts with $u = \ln x$, $dv = \sqrt{x}$, $du = dx/x$, $v = 2x^{3/2}/3$:

$$\int \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C.$$

(Check by differentiating.) Now we can evaluate the definite integral:

$$\int_1^4 \sqrt{x}\ln x\,dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2}\Big|_1^4 = \frac{2}{3}(4^{1/2})^3\ln 4 - \frac{4}{9}(4^{1/2})^3 - \frac{2}{3}1^{3/2}\ln 1 + \frac{4}{9}1^{3/2} = \frac{32}{3}\ln 2 - \frac{32}{9} + \frac{4}{9}.$$

(Numerically I get an answer of 4.2825.)

(b) We can often find the integrals of inverse functions by multiplying by 1 and integrating by parts. Set $u = \cos^{-1}x$, $dv = 1$, $du = -1/\sqrt{1-x^2}$, $v = x$:

$$\int \cos^{-1}x\,dx = \int 1\cdot\cos^{-1}x\,dx = x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}}\,dx.$$

We can evaluate the latter integral above either by the trig substitution $u = \sin x$ or by the algebraic substitution $u = 1 - x^2$, $du = -2x\,dx$, $-du/2 = x\,dx$:

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int\frac{du}{\sqrt{u}} = -\frac{1}{2}\int u^{-1/2}\,du = -u^{1/2} + C = -(1-x^2)^{1/2} + C.$$

(Check by differentiating.) Putting it all together,

$$\int \cos^{-1}x\,dx = x\cos^{-1}x - (1-x^2)^{1/2} + C.$$

As usual, check the final answer by differentiating.

(c) The power of sin is odd, so we try a substitution of the form $u = \cos(5t)$, $du = -5\sin(5t)\,dt$, $-du/5 = \sin(5t)\,dt$:

$$\int \cos^5(5t)\sin(5t)\,dt = -\frac{1}{5}\int u^5\,du = -\frac{1}{30}u^6 + C = -\frac{1}{30}\cos^6(5t) + C.$$

Check by differentiating.

(d) The integrand is defined and continuous on the interval $[0,\pi/3]$ so the integral is proper. The power of tan is odd so we try to evaluate the indefinite integral by the substitution $u = \sec\theta$, $du = \sec\theta\tan\theta\,d\theta$:

$$\int \tan\theta\sec^3\theta\,d\theta = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sec^3\theta + C.$$

Check by differentiating. The definite integral can now be evaluated:

$$\int_0^{\pi/3}\tan\theta\sec^3\theta\,d\theta = \frac{1}{3}\sec^3\theta\Big|_0^{\pi/3} = \frac{1}{3}\sec^3\frac{\pi}{3} - \frac{1}{3}\sec^3 0 = \frac{7}{3}.$$

(e) To reduce the power of x in the denominator, make the substitution $u = x^2$, $du = 2x\,dx$, $du/2 = x\,dx$, $x^4 = u^2$:

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u + C = \frac{1}{2}\sin^{-1}(x^2) + C.$$

Check by differentiating.

(f) This is a case where a trig substitution really helps. Let $x = 5\tan\theta$, $dx = 5\sec^2\theta\,d\theta$:

$$\int \frac{dx}{x^2\sqrt{x^2+25}} = \int \frac{5\sec^2\theta\,d\theta}{25\tan^2\theta\cdot 5\sec\theta} = \frac{1}{25}\int \frac{\cos\theta\,d\theta}{\sin^2\theta}.$$

Now make the substitution $u = \sin\theta$, $du = \cos\theta\,d\theta$:

$$\int \frac{dx}{x^2\sqrt{x^2+25}} = \frac{1}{25}\int \frac{du}{u^2} = -\frac{1}{75}u^{-3} + C = -\frac{1}{25}(\sin\theta)^{-1} + C.$$

Note that $(\sin\theta)^{-1}$ means $1/\sin\theta$ not $\arcsin\theta$. In the right triangle with $\tan\theta = x/5$ we have $O = x$, $A = 5$, and $H = \sqrt{x^2+25}$, so $\sin\theta = x/\sqrt{x^2+25}$. Putting it all together we get

$$\int \frac{dx}{x^2\sqrt{x^2+25}} = -\frac{(x^2+25)^{1/2}}{25x} + C.$$

Check by differentiating.

(g) **Solution 1.** The integral is improper of Type I because the domain over which we are integrating is unbounded The function $f(x) = \dfrac{2}{\sqrt{x^2-1}}$ is defined and continuous for all $x \geq 3$ (we just have to check that $x^2 - 1 > 0$ on the interval $[3,\infty)$, and it follows that the square root is defined and continuous and never 0, so the reciprocal is defined and continuous), so the integral is not improper of Type II. Therefore we can calculate the value of the definite integral if we can calculate

$$\lim_{t\to\infty}\int_3^t \frac{2}{\sqrt{x^2-1}}\,dx.$$

To evaluate the indefinite integral we make the trig substitution $x = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$:

$$\int \frac{2}{\sqrt{x^2-1}}\,dx = \int \frac{2}{\tan\theta}\,\sec\theta\tan\theta\,d\theta = \int 2\sec\theta\,d\theta = 2\ln|\sec\theta + \tan\theta| + C.$$

Reversing the substitution with $\sec\theta = x$, $\tan\theta = \sqrt{x^2-1}$ we obtain

$$\int \frac{2}{\sqrt{x^2-1}}\,dx = 2\ln|x + \sqrt{x^2-1}| + C.$$

Check by differentiating. Evaluating the definite integral,

$$\int_3^t \frac{2}{\sqrt{x^2-1}}\,dx = 2\ln|t + \sqrt{t^2-1}| - 2\ln|3+\sqrt{8}|.$$

Taking the limit as $t \to \infty$ we have $t + \sqrt{t^2-1} \to \infty$ so $\ln|t + \sqrt{t^2-1}| \to \infty$ and the integral is divergent.

Solution 2. For x large, $x^2 - 1 \approx x^2$ so the integrand is approximately $2/x$, the integral of which is divergent on $[3,\infty)$. So it seems that our integral is divergent. Let's prove it using the comparison test. We want to compare from below by some multiple of $1/x$; let's try $2/x$. Then for $x > 1$ we have

$$0 < \frac{2}{x} \leq \frac{2}{\sqrt{x^2-1}} \Leftrightarrow 0 < x^2 - 1 < x^2$$

and the latter is always true for $x \geq 3$. Since the given integral is bounded from below by a divergent integral, the given integral must also be divergent.

(h) **Solution 1.** The integrand tends to infinity as $x \to 4^-$ (and is defined and continuous for all other values of x on the domain over which we are integrating, i.e., $[0,4)$) so the integral is improper of Type II. To evaluate the integral we take the limit

$$\int_0^4 \frac{1}{\sqrt{4-x}}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{\sqrt{4-x}}\,dx.$$

To evaluate the improper integral we make the substitution $u = 4-x$, $-du = dx$:

$$\int \frac{1}{\sqrt{4-x}}\,dx = -\int u^{-1/2}\,du = -2u^{1/2} + C = -2(4-x)^{1/2} + C.$$

Evaluating the definite integral,

$$\int_0^t \frac{1}{\sqrt{4-x}}\,dx = -2(4-t)^{1/2} + 2(4-0)^{1/2} = -2(4-t)^{1/2} + 4.$$

Taking the limit as $t \to 4$ we obtain the answer

$$\int_0^4 \frac{1}{\sqrt{4-x}}\,dx = \lim_{t\to 4} -2(4-t)^{1/2} + 4 = -2(4-4)^{1/2} + 4 = 4.$$

In this question the comparison test would just tell us that the integral is convergent, and we would still have to evaluate the integral as we did above.

Solution 2.

6. (This problem is rather difficult.)

(a) Informally, $a_n > 5$ implies that $a_n + 5 > 10$ which implies that $a_{n+1} = (a_n+5)/2 > 5$ for all $n \geq 1$. More formally, we should prove the result by induction, but let's not worry about that.

(b) By the above argument, $a_n > 5$ for all $n \geq 1$, so adding a_n to both sides, $a_n + 5 < 2a_n$ which implies that $a_{n+1} = (a_n+5)/2 < a_n$ for all $n \geq 1$, i.e., that the sequence is decreasing.

(c) The sequence is decreasing and bounded below, so by the monotone sequence theorem the sequence has a limit. But all we know about the value of the limit from the monotone sequence theorem is that it is 5 or larger. Numerically experimenting with a few values of a_n we have $a_1 = 10$, $a_2 = 7.5$, $a_3 = 6.25$, $a_4 = 5.625$, and so on, with the difference between a_n and 5 halving at each step. To prove that, we subtract 5 from both sides of the equality $a_{n+1} = (a_n+5)/2$ to obtain $a_{n+1} - 5 = (a_n - 5)/2$. It follows that $a_n = 5 + 5/2^{n-1}$ (which could be proven by induction but again we'll not go there), so taking the limit as $n \to \infty$ we obtain 5 as the limit of the sequence.

7. (a) The series is geometric with $a = 1$ and $r = \dfrac{\sin(\pi/6)}{\sin(\pi/3)}$, so the series converges if and only if $-1 < r < 1$. There are three ways to check the value of r:

Direct evaluation. By the geometry of the equilateral triangle, the right triangle with $A = \sqrt{3}$, $O = 1$, $H = 2$ gives $\sin(\pi/6) = 1/2$. Similarly we obtain $\sin(\pi/3) = \sqrt{3}/2$. Therefore $r = 1/\sqrt{3} < 1$; also clearly $r > -1$, so $-1 < r < 1$ and the series is convergent.

Calculator approximation. Make sure your calculator is in radian mode! The number that your calculator will give, something like 0.5774, is far enough from 1 that you can be confident that $0 < r < 1$, and so the series is convergen. (If r were closer to 1 you would have to consider whether the amount of accuracy that your calculator gives is enough to tell for sure whether $r > 1$ or $r < 1$.)

Estimation. Since sin is non-negative and increasing on $[0, \pi/2]$ it follows that $0 < \sin(\pi/6) < \sin(\pi/3)$; dividing through by $\sin(\pi/3)$ we have $0 < \dfrac{\sin(\pi/6)}{\sin(\pi/3)} < 1$.

(b) The series is alternating so we use the alternating series test. Let f be the function on the interval $[0,\infty)$ defined by $f(x) = 1/(7x+3)$. Then f is continuous and differentiable on its domain and $f'(x) = -7(7x+3)^{-2}$ so the function is decreasing. Furthermore the limit of $f(x)$ as $x \to \infty$ is 0 as we can see by dividing through by the highest power of x in the definition of f. Those properties carry over to the sequence $1/(7n+3)$: it decreases to 0, so by the alternating series test the given series is convergent.

(c) For n large, the highest powers of n dominate the numerator and denominator, so the terms are approximately $n^4/n^5 = 1/n$. Since the harmonic series with terms $1/n$ diverges, we expect that the given series diverges, which we attempt to show by the comparison test. We try to compare from below by $1/n$. For $n \geq 2$ we have

$$\frac{1}{n} \leq \frac{n^4-n^3-n}{n^5-n^2-n} \Leftrightarrow n^5-n^2-n \leq n^5-n^4-n^2 \Leftrightarrow n^4 < n$$

which, unfortunately, is false for large n. Let's try comparing with $0.5/n$ instead:

$$\frac{0.5}{n} \leq \frac{n^4-n^3-n}{n^5-n^2-n} \Leftrightarrow n^5-n^2-n \leq 2n^5-2n^4-2n^2 \Leftrightarrow 2n^4+n^2 \leq n^5+n$$

which is true for n large enough (because $\lim_{n\to\infty}(n^5+n)/(2n^4+n^2) = \infty$ so the value of fraction must be greater than 1 for n large enough). Therefore the given series is bounded below for large n by the divergent series with terms $0.5/n$, so the given series is divergent by the comparison test.

You could also use the limit comparison test, but it amounts to essentially the same thing.

(d) This looks like a candidate for the ratio test. The terms of the series are positive so we don't need to take absolute values. We have

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\sqrt{(n+1)!}/2^{n+1}}{\sqrt{n!}/2^n} = \lim_{n\to\infty}\frac{\sqrt{(n+1)!}}{\sqrt{n!}}\frac{2^n}{2^{n+1}} = \lim_{n\to\infty}\sqrt{\frac{(n+1)!}{n!}}\frac{1}{2} = \frac{1}{2}\lim_{n\to\infty}\sqrt{n+1} = \infty$$

so the series is divergent by the ratio test.

You could have also used the test for divergence, but it is a little tricky to show that the limit of terms of the series is not 0. Try it!

(e) This looks like a job for the integral test. In the following indefinite integral, make the substitution $u = \ln x$, $du = dx/x$ to obtain

$$\int \frac{1}{x(\ln x)^e}\,dx = \int u^{-e}\,du = \frac{1}{-e+1}u^{-e+1} + C = \frac{1}{-e+1}(\ln x)^{-e+1} + C.$$

It follows that the integral

$$\int_3^\infty \frac{1}{x(\ln x)^e}\,dx = \lim_{t\to\infty}\frac{1}{1-e}\frac{1}{(\ln x)^{e-1}} - \frac{1}{1-e}(\ln 3)^{1-e} = \frac{1}{e-1}(\ln 3)^{1-e}$$

is convergent (the limit is 0 because $e-1>0$ so the denominator of the fraction in the limit tends to ∞). Therefore by the integral test the given series is convergent.

8. (This problem is rather difficult, and uses material that was not covered thoroughly in this term's course.) The given series can be rewritten $f(x) = -x\cdot\frac{1}{1-(2x^2)}$ which can be expanded into a geometric series:

$$\frac{1}{1-(2x^2)} = 1+(2x^2)+(2x^2)^2+(2x^2)^3+\cdots+(2x^2)^k+\cdots = 1+2x^2+4x^4+8x^6+\cdots+2^kx^{2k}+\cdots$$

so

$$f(x) = -x-2x^3-4x^5-8x^7-\cdots-2^kx^{2k+1}-\cdots.$$

By the ratio test the series is convergent when

$$1 > \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|2x^2\right| = 2x^2,$$

i.e., when $x^2 < \frac{1}{2}$, i.e., when $-1/\sqrt{2} < x < 1/\sqrt{2}$. The series is divergent when $x^2 > \frac{1}{2}$, i.e., when $x < -1/\sqrt{2}$ or $x > 1/\sqrt{2}$. The radius of convergence is $1/\sqrt{2}$.

To find the interval of convergence we must also test the endpoints. When $x = 1/\sqrt{2}$ the series becomes

$$f\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot 2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 - \cdots - \frac{1}{\sqrt{2}} \cdot 2^k \cdot \left(\frac{1}{\sqrt{2}}\right)^{2k} - \cdots = -\frac{1}{\sqrt{2}}(1+1+1+\cdots)$$

which diverges. Similarly $f(-1/\sqrt{2})$ diverges. Therefore the interval of convergence is $(-1/\sqrt{2}, 1/\sqrt{2})$.

You could have also found a Taylor series for f centered at any point a where the function is defined (i.e., anywhere but $a = \pm 1/\sqrt{2}$) but the calculations would be somewhat more difficult.

9. Calculating derivatives of $f(x) = \ln(1+x)$ we have

$$\begin{aligned} f^{(0)}(x) &= \ln(1+x) \\ f^{(1)}(x) &= (1+x)^{-1} \\ f^{(2)}(x) &= (-1)(1+x)^{-2} \\ f^{(3)}(x) &= (-1)(-2)(1+x)^{-3} \\ &\cdots \\ f^{(n)}(x) &= (-1)(-2)\cdots(-(n-1))(1+x)^{-n} \\ &\cdots . \end{aligned}$$

Evaluating each of the derivatives at 0 we have

$$\begin{aligned} f^{(0)}(0) &= \ln(1+0) = 0 \\ f^{(1)}(0) &= (1+0)^{-1} = 1 \\ f^{(2)}(0) &= (-1)(1+0)^{-2} = -1 \\ f^{(3)}(0) &= (-1)(-2)(1+x)^{-3} = 2 \\ &\cdots \\ f^{(n)}(0) &= (-1)(-2)\cdots(-(n-1))(1+x)^{-n} = (-1)^n(n-1)! \\ &\cdots . \end{aligned}$$

Dividing the n^{th} term by $n!$,

$$\begin{aligned} \frac{f^{(0)}(0)}{0!} &= \frac{0}{0!} = 0 \\ \frac{f^{(1)}(0)}{1!} &= \frac{1}{1!} = 1 \\ \frac{f^{(2)}(0)}{2!} &= -\frac{1}{2!} = -\frac{1}{2} \\ \frac{f^{(3)}(0)}{3!} &= \frac{2}{3!} = \frac{1}{3} \\ &\cdots \\ \frac{f^{(n)}(0)}{n!} &= (-1)^n \frac{(n-1)!}{n!} = (-1)^n \frac{1}{n} \\ &\cdots . \end{aligned}$$

Therefore the Maclaurin series for $f(x) = \ln(1+x)$ is

$$f(x) = 0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots + (-1)^n \frac{1}{n} x^n + \cdots$$

from which the first three nonzero terms can be read off.