

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 111-001-002-003-004-L01-L02-S01
Final Examination 200510

Time: 3 hours

Name: _____

Instructors:

Student #: _____

Dr. R. McIntosh (001)

Section: _____

Dr. M. Torres (002)

Dr. H. Weston (003/004)

Dr. I. Husain (L01/L02)

Dr. N. Fuller (S01)

(Marks)

Show all of your work on the pages of this examination paper. Use the backs of the pages if necessary. Do not hand in any scratch paper. Only Sharp EL-510RB calculators are permitted.

(12) 1. Find the derivative of the following functions. Do not simplify.

(a) $y = e^x \sin^{-1}(3x)$

(b) $y = \frac{1}{\tan^{-1} x}$

(c) $y = (x + \sin x)^x$

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- (8) 2. Evaluate the following limits if they exist.

(a) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

(b) $\lim_{x \rightarrow 0^+} \frac{1 - \ln x}{e^{1/x}}$

- (8) 3. Let $f(x) = e^{x \tan^{-1} x}$ for $x \geq 0$.

(a) Show that f is one-to-one.

(b) Evaluate $(f^{-1})'(1)$.

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- (6) 4. Find the volume of the solid obtained by rotating, about the y -axis, the region bounded by $y = 0$, $y = \sin x$, $x = \pi/4$ and $x = \pi$.

- (32) 5. Evaluate the following integrals.

(a) $\int_1^4 \sqrt{x} \ln x \, dx$

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(b) $\int \cos^{-1} x \, dx$

(c) $\int \cos^5(5t) \sin(5t) \, dt$

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(d) $\int_0^{\pi/3} \tan \theta \sec^3 \theta \, d\theta$

(e) $\int \frac{x}{\sqrt{1-x^4}} \, dx$

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(f) $\int \frac{dx}{x^2 \sqrt{x^2 + 25}}$

(g) $\int_3^{\infty} \frac{2}{\sqrt{x^2 - 1}} dx$

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(h) $\int_0^4 \frac{dx}{\sqrt{4-x}}$

(9) 6. Let $a_1 = 10$ and $a_{n+1} = (5 + a_n)/2$ for $n \geq 1$.

(a) Show that $a_n > 5$ for all $n \geq 1$.

(b) Show that the sequence $\{a_n\}$ is decreasing.

(c) Find the limit of $\{a_n\}$.

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(15) 7. Test the following series for convergence.

(a)
$$\sum_{n=1}^{\infty} \left(\frac{\sin(\pi/6)}{\sin(\pi/3)} \right)^n$$

(b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{7n+3}$$

(c)
$$\sum_{n=2}^{\infty} \frac{n^4 - n^3 - 1}{n^5 - n^2 - n}$$

(d)
$$\sum_{n=0}^{\infty} \frac{\sqrt{n!}}{2^n}$$

(e)
$$\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^e}$$

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- (6) 8. Find a power series representation for the function $f(x) = \frac{x}{2x^2 - 1}$ and give its interval of convergence.

- (4) 9. Find the first three nonzero terms of the Maclaurin series for $f(x) = \ln(1 + x)$.