# MATH 111-002 200530 Problem Set 1

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Due: Thursday, September 22, 2005

Please hand this problem set in at the end of the tutorial on Thursday, September 22, 2005. Each question is worth 1 mark out of 10.

1. Find a formula for the inverse of each of the functions

   (a)
   $$f(x) = \frac{2x - 1}{-3x + 2} \tag{1}$$

   (b)
   $$g(x) = (x + 2)^5 \tag{2}$$

2. Consider the function
   $$h(x) = x^3 + 2x + 3. \tag{3}$$
   (a) Show that $h(x)$ is one-to-one.
   (b) Find $(h^{-1})'(6)$.

3. Differentiate the functions

   (a)
   $$k(x) = (x^3 + 1)e^x \tag{4}$$

   (b)
   $$m(x) = \sin(e^{x^2}) \tag{5}$$

4. Evaluate the integrals

   (a)
   $$\int_0^3 e^{2x}\,dx \tag{6}$$

   (b)
   $$\int x e^{2x^2}\,dx \tag{7}$$

5. Find the equation of the tangent line to the curve $y = e^x \sin(x)$ at the point $x = \pi$.

6. Find the values of $\lambda$ for which $y = e^{\lambda x}$ satisfies the differential equation $y'' = y' + 6y$.

7. Find the limit
$$\lim_{x \to \infty} \frac{2e^x - 1}{-3e^x + 2} \tag{8}$$

8. The radioactive isotope Carbon 14 has a half life of $5.73 \times 10^3$ years. How long will it take for a sample of 0.3 micrograms of Carbon 14 to decay to 0.1 micrograms of Carbon 14?

9. Find the hundredth derivative of the function $p(x) = (x + 50)e^{-x}$.

10. Show that
$$e^x \geq 1 + x + \frac{x^2}{2} \tag{9}$$
for $x > 0$. (Hint: see the questions near the end of section 7.2 in the textbook.) Use that result to estimate $\int_0^1 e^{x^2}$. What is the error in your estimate?

Questions which you don't have to hand in, but which you should do for practice, are

C-level 7.1: 20, 21, 25–30, 39–42 (also show that $f$ is one-to-one); 7.2: 15–16, 23–28, 29–42, 71–78

B-level 7.1 23–24; 7.2: 45, 52, 58, 61–62, 83

A-level 7.1: 45–48; 7.2: 84-87