

MATH 111 002 200530 Problem Set 5

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Due: Friday, October 28, 2005, at the beginning of the lecture

Please hand the following problems in. The last two are more difficult than the others, as usual.

1. (2 marks) Evaluate the following integrals.

$$(a) \int \frac{\sqrt{x^2 - 16}}{x^4} dx \quad (b) \int_0^{\sqrt{5}} \frac{x}{\sqrt{9 - x^2}} dx \quad (c) \int \frac{x^2}{(a^2 - x^2)^{5/2}} dx \quad (d) \int \frac{x^2}{(a^2 + x^2)^{5/2}} dx$$

2. (2 marks) Evaluate the following integrals. Use Example 8 on page 523 where necessary.

$$(a) \int \frac{dx}{\sqrt{16x^2 - 9}} \quad (b) \int_0^{2/3} x\sqrt{4 - 9x^2} dx \quad (c) \int_0^1 \sqrt{4x^2 + 1} dx \quad (d) \int \frac{dx}{(a^2x^2 - b^2)^{5/2}}$$

3. (2 marks) Evaluate the following integrals. Use (b) and (c) to answer (d).

$$(a) \int \frac{dx}{(x^2 + 2x + 2)^4} \quad (b) \int \frac{3}{\sqrt{t^2 - 6t + 1}} dt \quad (c) \int \frac{(t - 3)}{\sqrt{t^2 - 6t + 1}} dt \quad (d) \int \frac{t}{\sqrt{t^2 - 6t + 1}} dt$$

4. (2 marks) Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent.

$$(a) \int_0^{\infty} \frac{x}{1 + x + x^2} dx \quad (b) \int_0^{\infty} \frac{1}{t^2 + 4t + 3} dt \quad (c) \int_0^2 \frac{1}{\sqrt{4 - x^2}} dx \quad (d) \int_0^3 (z - 2)^{-1/3} dz$$

5. (1 mark) Taking reduction formula 97 in the table at the back of the textbook from 0 to t gives the the following formula for definite integrals:

$$\int_0^t x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} \Big|_0^t - \frac{n}{a} \int_0^t x^{n-1} e^{ax} dx.$$

Assume $a < 0$ and take the limit as $t \rightarrow \infty$ to obtain a reduction formula for $\int_0^{\infty} x^n e^{ax} dx$. Use your formula to evaluate $\int_0^{\infty} x^2 e^{-3x} dx$.

6. (1 mark) Evaluate $\int_0^{\pi/2} \frac{\sin t}{\sqrt{1 + \cos^2 t}} dt$.

Please do the following problems from the textbook. You do not need to hand in your solutions to these problems!

8.3 C-level: 1–24, 33; B-level: 25–29, 32(a), 34–35, 38; A-level: 30, 39

8.8 C-level: 1–3, 5–14, 16–38, 41–42, 49–54; B-level: 39–40, 55–59, 61, 62, 64–65 (see exercises 6.4.29 and 6.4.30), 69; A-level: 60, 66, 68, 71–78