MATH111-002 200530 Problem Set 6

Edward Doolittle

Due: Monday, November 14, 2005, at the beginning of the lecture

Please hand the following problems in. The last two are more difficult than the others, as usual.

1. (4 marks) Use the disk or washer method to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, a typical area element of a cross section, and a typical disk or washer.

   (a) $y = x^{2/3}$, $x = 4$, $y = 0$; about the x-axis

   (b) $y = x^{2/3}$, $y = 8$, $x = 0$; about the y-axis

   (c) $y = x^{2/3}$, $x = 4$, $y = 0$; about the line $y = -1$

   (d) $y = x^{2/3}$, $y = 8$, $x = 0$; about the line $x = -2$

2. (2 marks) Use the shell method to find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, a typical area element of a cross section, and a typical shell.

   (a) $y = x^3$, $y = 8$, $x = 0$, about y-axis.

   (b) $y = x^3$, $y = 8$, $x = 0$, about x-axis.

3. (2 marks) Find the volume of the solid obtained by rotating the the region bounded by the given curves about the specified line. State which method you are using (disk/washer or shell) and sketch the region, a typical area element of a cross section, and a typical shell.

   (a) $y = \sin x$, $y = 0$, $0 \le x \le \pi$; about the y-axis

   (b) $y = \sin x \cos x$, $y = 0$, $0 \le x \le \pi/2$; x-axis

4. (1 mark) Use the shell method to find the volume of the ellipsoid of revolution generated by rotating the ellipse $4x^2 + 9y^2 = 1$ about the x-axis.

5. (1 mark) **Archimedes' tombstone.** Consider the unit circle $x^2 + y^2 = 1$ and the square bounded by the lines $x = \pm 1$, $y = \pm 1$. Rotating these figures about the y-axis gives a sphere and a cylinder respectively; we say that the sphere is *inscribed* in the cylinder. Archimedes requested that the resulting three-dimensional figure be carved on his tombstone as a monument to the discovery of which he was most fond: a proof that the volume of a sphere is $2/3$ the volume of the circumscribed cylinder. (A similar relationship holds for surface areas.)

   Following Archimedes' arguments we first find the relationship between volumes of the cylinder and the sphere. Pick a point (x, y) on the circle. Calculate the volume V_1 of the shell element generated by the rectangle between the x-axis and the point (x, y) with width dx. Calculate the volume V_2 of the corresponding washer generated by the rectangle between the point (x, y) and the line $x = 1$ with width dy. Show that $V_1 = 2V_2$. (Hint: $x^2 + y^2 = 1 \implies 2x\,dx + 2y\,dy = 0$.) It follows that the volume of the sphere is twice the volume of the region outside the sphere but inside the cylinder; so the volume of the sphere is 2/3 the volume of the cylinder. Since we know the volume of the cylinder we now know the volume of the sphere, without integrating.

Please do the following problems from the textbook. You do not need to hand in your solutions to these problems!

6.2 C-level: 1–36, 41–44; B-level: 47–49, 61, 69; A-level: 45–46, 65, 70

6.3 C-level: 1–26, 29–32; B-level: 37–45; A-level: 46

There are also volume of revolution questions scattered throughout chapters 7 and 8, e.g., 7.2.81–82, 7.4.79–80, 7.5.72, 8.1.55–58, 8.2.59–62, 8.3.41. You should try a few of those. Problem 8.1.65 (B-level) shows that the shell method and disk method are essentially interchangeable. Problem 8.8.63 gives a peculiar example.