MATH111-002 200530 Problem Set 7

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Due: Friday, November 18, 2005, at the beginning of the lecture

Please hand the following problems in. The last two are more difficult than the others, as usual.

1. (1 mark) List the first five terms of the sequence.

   (a) $a_n = \dfrac{(-5)^n}{n!}$ (b) $b_n = 2(1-x)^n$

2. (1 mark) Find a formula for the general term a_n of the sequence.

   (a) $3, 7, 11, 15, 19, \ldots$ (b) $-\dfrac{1}{2}, \dfrac{4}{4}, -\dfrac{9}{8}, \dfrac{16}{16}, -\dfrac{25}{32}, \ldots$

3. (1 mark) Determine whether or not the sequence converges. If it converges, find the limit

   (a) $a_n = \dfrac{3^n}{4^{1-n}}$ (b) $b_n = n^e e^{-n}$

4. (1 mark) Determine whether or not the sequence is increasing, decreasing, or neither. Is the sequence bounded?

   (a) $a_n = \dfrac{1}{n-(1/2)}$ (where $n = 1, 2, \ldots$) (b) $b_n = \dfrac{2n}{n^2+4}$

5. (1 mark) Find 5 partial sums of the following series.

   (a) $a_n = \dfrac{(-5)^n}{n!}$ (b) $b_n = 2(1-x)^n$

6. (2 marks) Determine whether the following series are convergent or divergent. If a series converges, find its sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} 5\left(\frac{3}{2}\right)^{n-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{10^{n-1}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+5}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+4^n}{8^n}$

7. (2 marks) Find the values of x for which the following series converge. Find the sum of the series for those values of x.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{3x^n}{4^n}$ (b) $\displaystyle\sum_{n=0}^{\infty} 5^n \left(\frac{x}{4}\right)^n$ (c) $\displaystyle\sum_{n=0}^{\infty} 2(1-x)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{\tan^n x}{3^{n/2}}$

8. (1 mark) Show that the series $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$ is divergent.

Please do the following problems from the textbook. You do not need to hand in your solutions to these problems!

12.1 C-level: 3–22, 35, 54–60; B-level: 23–34, 36–40, 50–52, 61–65; A-level: 66, 71–72

12.2 C-level: 9–28, 41–45; B-level: 29–40, 46, 49, 53, 58, 63; A-level: 64–67