

# MATH111-002 200630 Problem Set 2

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Quizzed: Thursday, September 28, 2006

Problems from this problem set will be used in the quiz on Thursday, September 28, 2006.

1. Find the exact value of each of the following expressions. Give your answer in terms of a rational number (i.e., either as a fraction  $p/q$  where  $p$  and  $q$  are integers, or as a terminating decimal, or as a repeating decimal).

(a)  $2\log_{10} 3 - \log_{10} 90$

(b)  $\ln(e^{2^3})$

2. Solve each of the following equations for  $x$ . Express your answer as a number correct to four decimal places.

(a)  $\ln(5x - 2) = -7$

(b)  $2^{3x+4} = 10$

3. Differentiate each of the following functions.

(a)  $f(x) = \sqrt{x \ln x}$

(b)  $g(\theta) = 2^{\sin \theta}$

4. Use logarithmic differentiation to find each of the following derivatives:

(a)  $y = (2x^3 + 1)^4(x - 1)^{-1/2}$

(b)  $h(x) = \frac{\sin^2 x \cos^4 x}{(x^2 + 1)^3}$

5. Use implicit differentiation to find a formula for  $y'$  in terms of  $x$  and  $y$  if  $y^2 = e^{x^2+y^2}$ .

6. Evaluate the integral

$$\int_4^{16} \left( \sqrt{x} + \frac{3}{\sqrt{x}} \right)^2 dx.$$

7. When the flash in a camera is used, the camera immediately begins to recharge the flash's capacitor, which stores electric charge according to the formula  $Q(t) = Q_0(1 - e^{-t/a})$  where  $Q_0$  is the maximum charge capacity,  $t$  is the time in seconds since the process of charging the flash began, and  $a$  is a parameter depending on the camera.

(a) Find the inverse of this function and explain its meaning.

(b) How long will it take to recharge the capacitor to 75% of its maximum capacity if  $a = 3$ ?

8. Find an equation of the tangent to the curve  $y = e^x$  that is perpendicular to the line  $x + 2y = 5$ .

9. Find the limit  $\lim_{x \rightarrow 0^+} x^{\ln x}$  using the formulas  $x = e^{\ln x}$  and  $\lim_{x \rightarrow a} e^{f(x)} = e^{\lim_{x \rightarrow a} f(x)}$

10. Show that  $\lim_{n \rightarrow \infty} \left(1 + \frac{a}{n}\right)^n = e^a$  for any  $a > 0$ . Hint: you can either use formula 8 in section 7.4 of the textbook, or the result of problem 87 of section 7.4.

For practice, you should do the following additional problems:

C-level 7.3: 3–8, 9–12, 13–18, 20–22, 29–38, 47–49; 7.4: 2–36, 39–54, 57–58, 59–62 (do only one graph), 65–76.

B-level 7.3: 39–44, 51–71, 75; 7.4: 77, 81, 83–85

A-level 7.4: 87–88