

MATH111-002 200630 Problem Set 3

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Quizzed: Thursday, October 19, 2006

1. Simplify the following expressions:

(a) $\sec(\arcsin(4/5))$.

(b) $\sin(\arctan 2x)$.

2. Find the following limits:

(a) $\lim_{t \rightarrow 0} \frac{e^t - 2^t}{\tan t}$.

(c) $\lim_{x \rightarrow \infty} (1 - 5x) \sin(1/x)$.

(b) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3 \tan^2 x + x^5}$.

(d) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} + \frac{3}{x^2}\right)^x$.

3. Find the following derivatives:

(a) $f'(x)$ where $f(x) = \arcsin(e^x)$.

(c) $g''(t)$ where $g(t) = \cos^{-1}(3 - 2t)$.

(b) $h'(z)$ where $h(z) = \cot^{-1}(e^z) + \cot^{-1}(e^{-z})$.

(d) y' where $\tan^{-1}(xy) = 1 + x^2y^2$.

4. Find the following integrals. Evaluate to four decimal points where appropriate.

(a) $\int \frac{3}{t^2 + 9} dt$

(c) $\int \frac{x+4}{x^2+16} dx$

(b) $\int_{-\pi/4}^{\pi/4} \frac{\sin x}{\sqrt{1 - \cos^2 x}} dx$

(d) $\int \frac{2x^2}{\sqrt{1-x^6}} dx$

5. The limit $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x)$ is said to be of the form $\infty - \infty$.

(a) One way to do the limit is to multiply by the conjugate radical $\frac{\sqrt{x^2 + 2x} + x}{\sqrt{x^2 + 2x} + x}$. Do so, and evaluate the resulting limit.

(b) Another way would be to convert it into a limit of the form $\infty \cdot 0$ by taking out a factor of x . Do so, and then use L'Hôpital's rule to evaluate the resulting limit.

6. Evaluate $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \tan(t^2) dt$.

7. Solve for a :

$$\lim_{x \rightarrow \infty} \left(\frac{x+a}{x-a} \right)^x = e.$$

The following problems from the textbook may show up on midterm exams or on the final exam.

7.5 C-level: 1–8, 12–13, 22–33, 38–48, 51–54 (just do one or two), 59–70; B-level: 9–11, 14, 17–21, 34–37, 43–46, 48–50, 57–58; A-level: 74–80

7.7 C-level: 1–38, 67–72 (just do one or two; L'Hôpital's rule helps find asymptotes); B-level: 39–62, 73–75, 78–80, 83, 87–89; A-level: 81–82, 84–86, 93