

MATH111-002 200630 Problem Set 5

Edward Doolittle

Test: Thursday, November 2, 2006

The following problems from sections 8.3 and 8.4 of the textbook may appear on Midterm Test 2 on Thursday, November 2, 2006. The last two problems are more difficult than the others, as usual.

1. Evaluate the following integrals.

$$(a) \int \frac{\sqrt{x^2 - 16}}{x^4} dx \quad (b) \int_0^{\sqrt{5}} \frac{x}{\sqrt{9 - x^2}} dx \quad (c) \int \frac{x^2}{(a^2 - x^2)^{5/2}} dx \quad (d) \int \frac{x^2}{(a^2 + x^2)^{5/2}} dx$$

2. Evaluate the following integrals. Use Example 8 on page 523 where necessary.

$$(a) \int \frac{dx}{\sqrt{16x^2 - 9}} \quad (b) \int_0^{2/3} x\sqrt{4 - 9x^2} dx \quad (c) \int_0^1 \sqrt{4x^2 + 1} dx \quad (d) \int \frac{dx}{(a^2x^2 - b^2)^{5/2}}$$

3. Evaluate the following integrals. Use (b) and (c) to answer (d).

$$(a) \int \frac{dx}{(x^2 + 2x + 2)^4} \quad (b) \int \frac{3}{\sqrt{t^2 - 6t + 1}} dt \quad (c) \int \frac{(t - 3)}{\sqrt{t^2 - 6t + 1}} dt \quad (d) \int \frac{t}{\sqrt{t^2 - 6t + 1}} dt$$

4. Evaluate the following integrals by partial fractions.

$$(a) \int \frac{r^2}{r + 4} dr \quad (b) \int_0^1 \frac{x^3 - 4x - 10}{x^2 - x - 6} dx \quad (c) \int \frac{ds}{s^2(s - 1)^2} \quad (d) \int \frac{x^3}{x^3 + 1} dx.$$

5. Evaluate the following integrals by a rationalizing substitution followed by partial fractions.

$$(a) \int \frac{1}{1 - \sqrt{x + 2}} dx \quad (b) \int_0^1 \frac{1}{1 + \sqrt[3]{x}} dx \quad (c) \int \frac{\cos x}{\sin^2 x + \sin x} dx \quad (d) \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx$$

6. **The half-angle substitution.** This is one of the trickiest rationalizing substitutions ever invented, to be used only as a last resort. Let $t = \tan(x/2)$; show that

$$\cos x = \frac{1 - t^2}{1 + t^2}, \quad \sin x = \frac{2t}{1 + t^2}, \quad dx = \frac{2}{1 + t^2} dt,$$

and use those results to evaluate $\int \frac{dx}{3 - 5 \sin x}$ (which is cause for a last resort, by the way).

Please do the following problems from the textbook. They may appear on the midterm test or final exam.

8.3 C-level: 1–24, 33; B-level: 25–29, 32(a), 34–35, 38; A-level: 30, 39

8.4 C-level: 1–18, 25–27, 29, 32, 35, 39–42, 60–61; B-level: 19–24, 28, 30–31, 33–34, 36–38, 43–50, 53–54, 63–64; A-level: 55–59, 67–68