

MATH111-002 200630 Problem Set 6

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Quiz: Thursday, November 9, 2006

The following problems may appear on the quiz on Thursday, November 9, 2006. The last two are more difficult than the others, as usual.

1. Integrate each of the following functions.

(a) $\int \frac{1}{(x^2+2)^2} dx$ (b) $\int x^2 e^{-x^3} dx$ (c) $\int re^{r/3} dr$ (d) $\int \frac{\ln x}{x^3} dx$

2. Determine whether each of the following type I improper integrals is convergent or divergent. Evaluate those that are convergent.

(a) $\int_0^\infty \frac{1}{(x^2+2)^2} dx$ (b) $\int_{-\infty}^\infty x^2 e^{-x^3} dx$ (c) $\int_{-\infty}^0 re^{r/3} dr$ (d) $\int_1^\infty \frac{\ln x}{x^3} dx$

3. Determine whether each of the following type II improper integrals is convergent or divergent. Evaluate those that are convergent.

(a) $\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$ (b) $\int_0^3 (z-2)^{-1/3} dz$ (c) $\int_0^4 \frac{1}{x^2+x-6} dx$ (d) $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$

4. Use the Comparison Theorem to determine whether each of the following integrals is convergent or divergent.

(a) $\int_1^\infty \frac{2+e^{-x}}{x} dx$ (b) $\int_1^\infty \frac{x}{\sqrt{1+x^6}} dx$ (c) $\int_0^1 \frac{e^{-x}}{\sqrt{x}} dx$ (d) $\int_0^{\pi/2} \frac{dx}{x \sin x}$

5. Taking reduction formula 97 in the table at the back of the textbook from 0 to t gives the the following formula for definite integrals:

$$\int_0^t x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} \Big|_0^t - \frac{n}{a} \int_0^t x^{n-1} e^{ax} dx.$$

Assume $a < 0$ and take the limit as $t \rightarrow \infty$ to obtain a reduction formula for $\int_0^\infty x^n e^{ax} dx$. Use your formula to evaluate $\int_0^\infty x^2 e^{-3x} dx$. (You can use the fact that $\lim_{t \rightarrow \infty} t^n e^{at} = 0$ for $a < 0$, which can be proven using L'Hôpital's rule repeatedly.)

6. Find the value of the constant C for which the integral $\int_0^\infty \left(\frac{x}{x^2+1} - \frac{C}{3x+1} \right) dx$ converges. Evaluate the integral for this value of C .

Please do the following problems from the textbook. They may appear on the final exam.

8.5 C-level: 1–5, 7–26, 36–38, 40, 66; B-level: 6, 27–28, 30–35, 39, 41–52, 54–60, 68, 69; A-level: 61–65, 67, 70–81

8.8 C-level: 1–3, 5–14, 16–38, 41–42, 49–54; B-level: 39–40, 55–59, 61, 62, 64–65 (see exercises 6.4.29 and 6.4.30), 69; A-level: 60, 66, 68, 71–78