

MATH 111-002 200630 Problem Set 8

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Quiz: Thursday, November 23, 2006

The following problems from chapters 9.1 and 10.1 may appear on the quiz on Thursday, November 23.

1. Find the length of the following curves.

(a) $y = \frac{x^2}{2} - \frac{\ln x}{4}, 2 \leq x \leq 4$

(c) $y = \ln\left(\frac{e^x + 1}{e^x - 1}\right), a \leq x \leq b, a > 0$

(b) $y^2 = 4x, 0 \leq y \leq 2$

(d) $y = \ln(\cos x), 0 \leq x \leq \pi/3$

2. Find the arc length function $s(x)$ for the curve $y = x^3/3 + 1/(4x)$ with starting point $P_0(1, 7/12)$.

3. Verify that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

4. Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial value problem $y' + (\tan x)y = \cos^2 x, y(0) = -1$ on the interval $-\pi/2 < x < \pi/2$.

5. Which of the following functions are solutions of the differential equation $y'' + 2y' + y = 0$?

(a) $y = e^t$

(b) $y = e^{-t}$

(c) $y = te^{-t}$

(d) $y = t^2 e^{-t}$

6. For what values of r does the function $y = e^{rt}$ satisfy the differential equation $y'' + y' - 6y = 0$?

7. (a) Verify that all the members of the family of functions $y = 1/(x+c)$ are solutions of the differential equation $y' = -y^2$.

(b) Find a solution of the initial value problem $y' = -y^2, y(0) = 1/2$.

(c) Find a solution of the differential equation $y' = -y^2$ that is not of the form $y = 1/(x+c)$. (Hint: try graphing the family of functions.)

8. (a) Verify that all the members of the family of functions $y = (c - x^2)^{-1/2}$ are solutions of the differential equation $y' = xy^3$.

(b) Find a solution of the initial value problem $y' = xy^3, y(0) = 2$.

(c) Can you find a solution of the differential equation $y' = xy^3$ that is not of the form $y = (c - x^2)^{-1/2}$?

9. Sketch the curve with equation $x^{2/3} + y^{2/3} = 1$ and find its length.

10. Find the arc length of the curve $y = x^{4/3}$ from $(0, 0)$ to $(1, 1)$.

Please do the following problems from the textbook. They may appear on the final exam.

9.1 C-level: 1–12, 14–20; B-level: 29–34; A-level: 36–38

10.1 C-level: 1–8