MATH 111-002 200630 Problem Set 9

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Quiz: Thursday, November 30, 2006

The following problems from chapters 10.3 and 10.6 may appear on the quiz on Thursday, November 30.

1. Solve the following separable differential equations.

   (a) $\dfrac{dy}{dx} = \dfrac{y}{x}$ (b) $\dfrac{dy}{dx} = \dfrac{e^{2x}}{4y^3}$ (c) $\dfrac{dy}{dx} = y^2 \sin x$ (d) $\dfrac{du}{dt} = \dfrac{tu}{2\ln u}$

2. Solve the following initial value problems.

   (a) $\dfrac{dy}{dx} = \dfrac{y\cos x}{1+y^2}$, $y(0) = 1$ (b) $\dfrac{dP}{dt} = \sqrt{Pt}$, $P(1) = 2$ (c) $\dfrac{dy}{dt} = te^y$, $y(1) = 0$ (d) $x\dfrac{dy}{dx} + y = y^2$, $y(1) = -1$

3. Solve the following linear differential equations.

   (a) $y' = x + 5y$ (b) $x^2y' + 2xy = \cos^2 x$ (c) $y' = x\sin 2x + y\tan x$, $-\pi/2 < x < \pi/2$ (d) $t\ln t\, r' + r = te^t$

4. Solve the following initial value problems.

   (a) $y' = x + 5y$, $y(0) = 1$ (b) $t\,y' + 2y = t^3$, $t > 0$, $y(1) = 0$ (c) $2xy' + y = 6x$, $x > 0$, $y(4) = 20$ (d) $xy' - y/(x+1) = x$, $x > 0$, $y(1) = 0$

5. **Bernoulli differential equations.** A Bernoulli differential equation is of the form $\dfrac{dy}{dx} + P(x)y = Q(x)y^n$.

   (a) Show that a Bernoulli equation is linear if $n = 0$ or 1.
   (b) Show that, if $n \neq 0, 1$, the change of variables $u = y^{1-n}$ transforms the equation to a linear equation.
   (c) Using the above transformation, solve the equation $xy' + y = -xy^2$.
   (d) Use the above transformation to find another way to solve the separable equation $y' = -y^2$.
   (e) Solve $y' + y = xy^3$.

6. Find the orthogonal trajectories of the following families of curves.

   (a) $y = kx^2$ (b) $x^2 - y^2 = k$ (c) $y = (x+k)^{-1}$ (d) $y = ke^{-x}$

Please do the following problems from the textbook. They may appear on the final exam.

10.3 C-level: 1–18; B-level: 19–21, 27–30; A-level: 31–44

10.6 C-level: 1–20; B-level: 23–26; A-level: 27–36