MATH 111-002 200630 Problem Set 10

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Final Exam: Thursday, December 7, 2006

The following problems from Zill and Cullen (Ordinary Differential Equations) may appear on the final exam on Thursday, December 7, 2006.

1. Solve the following homogeneous differential equations.

   (a) $\frac{dy}{dx} = -\frac{x+y}{x}$ (b) $\frac{dy}{dx} = -\frac{y^2+yx}{x^2}$ (c) $\frac{dy}{dx} = \frac{x+3y}{3x+y}$ (d) $x\frac{dy}{dx} = y + \sqrt{x^2-y^2}$

2. Solve the following initial value problems.

   (a) $xy^2\frac{dy}{dx} = y^3 - x^3$, $y(1) = 2$ (b) $(x^2+2y^2)\frac{dx}{dy} = xy$, $y(-1) = 1$ (c) $\frac{dy}{dx} = \frac{x+ye^{y/x}}{xe^{y/x}}$, $y(1) = 0$ (d) $\frac{dy}{dx} = \frac{y/x}{\ln x - \ln y - 1}$, $y(1) = e$

3. Determine whether the following differential equations are exact.

   (a) $(2x-1)dx + (3y+7)dy = 0$

   (b) $(2x+y)dx - (x+6y)dy = 0$

   (c) $(x^3+y^3)dx + 3xy^2dy = 0$

   (d) $\left(1+\ln x+\frac{y}{x}\right)dx = (1-\ln x)dy$

4. Solve the following exact differential equations.

   (a) $(5x+4y)dx + (4x-8y^3)dy = 0$

   (b) $(\sin y - y\sin x)dx + (\cos x + x\cos y - y)dy = 0$

   (c) $(x^2-y^2)dx + (y^2-2xy)dy = 0$

   (d) $(3x^2y+e^y)dx + (x^3+xe^y-2y)dy = 0$

5. Solve the following initial value problems.

   (a) $(x+y)^2dx + (2xy+x^2-1)dy = 0$, $y(1) = 1$

   (b) $(e^x+y)dx + (2+x+ye^y)dy = 0$, $y(0) = 1$

   (c) $(4y+2t-5)dx + (6y+4t-1)dy = 0$, $y(-1) = 2$

   (d) $(y^2\cos x - 3x^2y - 2x)dx = (x^3 - 2y\sin x - \ln y)dy$, $y(0) = e$

6. **Integrating factors for exact equations.**

   (a) Show that the differential equation

   $$xy\,dx + (2x^2+3y^2-20)\,dy = 0$$

   is not exact.

   (b) Find an integrating factor $\mu(y)$ such that the equation

   $$\mu(y)xy\,dx + \mu(y)(2x^2+3y^2-20)\,dy = 0$$

   is exact, and solve the equation.

The following grab bag of differential equations and initial value problems is provided for extra practice. You should identify the type of equation, solve the equation, and then find the member of the solution family which satisfies the initial conditions (if applicable).

1. $(y^2+1)dx = y\sec^2 x\, dy$
2. $y(\ln x - \ln y)dx = (x\ln x - x\ln y - y)dy$
3. $y\, dx = (ye^y - 2x)dy$
4. $(6x+1)y^2\dfrac{dy}{dx} + 3x^2 + 2y^3 = 0$
5. $\dfrac{dx}{dy} = -\dfrac{4y^2+6xy}{3y^2+2x}$
6. $t\dfrac{dQ}{dt} + Q = t^4\ln t$
7. $(2x+y+1)y' = 1$
8. $\dfrac{dy}{dx} = \left(\dfrac{2y+3}{4x+5}\right)^2$
9. $(x^2+4)dy = (2x-8xy)dx$
10. $y' = 2y + x^2 + 5$
11. $-y\, dx + (x+\sqrt{xy})dy = 0$
12. $(2r^2\cos\theta\sin\theta + r\cos\theta)d\theta + (4r + \sin\theta - 2r\cos^2\theta)dr = 0$
13. $\dfrac{dr}{d\theta} + r\sec\theta = \cos\theta$
14. $(\tan x - \sin x\sin y)dx + \cos x\cos y\, dy = 0$
15. $\sin x\dfrac{dy}{dx} + (\cos x)y = 0,\ y\left(\dfrac{7\pi}{6}\right) = -2$
16. $(x+1)\dfrac{dy}{dx} + y = \ln x,\ y(1) = 10$
17. $xy' + y = e^x,\ y(1) = 2$
18. $\dfrac{dy}{dt} + 2(t+1)y^2 = 0,\ y(0) = -\dfrac{1}{8}$
19. $\left(\dfrac{3y^2-t^2}{y^5}\right)\dfrac{dy}{dt} + \dfrac{t}{2y^4} = 0,\ y(1) = 1$
20. $x^2\dfrac{dy}{dx} = y - xy,\ y(-1) = -1$