Math 111 Quiz 1 Solutions DRAFT

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1. First evaluate the indefinite integral. Let $u = 3x^2$. Then $du = 6x\,dx$ and

$$\int xe^{3x^2}\,dx = \int e^u \frac{du}{6} = \frac{1}{6}e^u + C = \frac{1}{6}e^{3x^2} + C.$$

   Check by differentiating. Now evaluate the definite integral:

$$\int_0^4 xe^{3x^2}\,dx = \frac{1}{6}e^{3x^2}\Big|_0^4 = \frac{1}{6}\left(e^{3\cdot 4^2} - e^{3\cdot 0^2}\right) = \frac{1}{6}\left(e^{48} - 1\right).$$

   My calculator says that the value of the integral is approximately 1.1695×10^{20}.

2. First, find the y-value of the point on the curve. When $x_0 = \pi$,

$$y_0 = e^{x_0}\cos(x_0) = e^{\pi/2}\sin(\pi/2) = e^{\pi/2}\cdot 1 = e^{\pi/2}.$$

   So the point on the curve is $(x_0, y_0) = (\pi/2, e^{\pi/2})$.

   Next find the slope of the tangent line through that point. By the product rule, the derivative of the function is

$$y' = e^x\cos(x) - e^x\sin(x).$$

   So the derivative for the given value of x_0 is

$$y'(x_0) = y'(\pi/2) = e^{\pi/2}\cos(\pi/2) - e^{\pi/2}\sin(\pi/2) = e^{\pi/2}\cdot 0 - e^{\pi/2}\cdot 1 = -e^{\pi/2}.$$

   That is the slope m of the tangent line.

   Since we know a point through which the line passes and the slope of the line, we can write down an equation in point-slope form $y - y_0 = m(x - x_0)$:

$$y - e^{\pi/2} = -e^{\pi/2}(x - \pi/2).$$

   The equation of the line can be put into another form if you so desire, but the answer is fine as it stands.