

MATH111-002 200630 Quiz 2 Solutions DRAFT

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1. We have learned two general approaches to differentiating functions of this type.

(a) **Replacement with the exponential function.** Using the rules for exponential functions, we can write

$$y = \sqrt{x^x} = x^{x/2} = (e^{\ln x})^{x/2} = e^{(x/2) \ln x}.$$

Now, using the chain rule for derivatives of exponential functions, i.e.,

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

with $u = (x/2) \ln x$, we have

$$y' = e^{(x/2) \ln x} \frac{d}{dx} ((x/2) \ln x) = \sqrt{x^x} ((1/2) \ln x + (1/2)).$$

Further simplification is possible but is not necessary.

(b) **Logarithmic differentiation.** Taking the logarithm of both sides and then applying logarithm laws,

$$\ln y = \ln \sqrt{x^x} = \frac{1}{2} \ln x^x = \frac{x}{2} \ln x.$$

Differentiating both sides gives

$$\frac{y'}{y} = \frac{1}{2} \ln x + \frac{1}{2}.$$

Multiplying through by $y = \sqrt{x^x}$ gives

$$y' = \sqrt{x^x} \left(\frac{1}{2} \ln x + \frac{1}{2} \right)$$

as in the previous answer.

2. The slope of the line $y + 2x = 7$ can be found by solving for y and looking at the coefficient of x :

$$y = -2x + 7$$

so the slope of the line is -2 . The slope of any perpendicular line must be the negative reciprocal, i.e.

$$m = -\frac{1}{-2} = \frac{1}{2}.$$

The slope of a tangent line to a curve is given by the derivative, so we must find the point(s) x_0 on the curve where

$$y'(x_0) = \frac{1}{2}$$

Differentiating the function defining the curve,

$$\frac{1}{x_0} = \frac{1}{2}$$

so $x_0 = 2$ is the only point where the curve has slope $1/2$. To find an equation of the tangent line at that point we find the corresponding $y_0 = y(x_0) = \ln 2$. In summary, the equation of the tangent line is given in point-slope form $y - y_0 = m(x - x_0)$ as

$$y - \ln 2 = \frac{1}{2}(x - 2)$$