MATH111-002 200630 Quiz 3 Solutions DRAFT

Edward Doolittle

October 24, 2006

1. The limit is of the form $\infty \cdot 0$ so we move one of the multiplicands into the denominator of a fraction and apply L'Hôpital's rule. I would say it is easier to handle reciprocals of algebraic functions than it is to handle reciprocals of trig functions, so let's try it that way first.

$$\lim_{x\to\infty}(1+3x)\sin(2/x) = \lim_{x\to\infty}\frac{\sin(2/x)}{(1+3x)^{-1}} = \lim_{x\to\infty}\frac{\cos(2/x)\cdot -2x^{-2}}{-(1+3x)^{-2}}.$$

Note that as $x \to \infty$, $2/x \to 0$ so $\cos(2/x) \to \cos(0) = 1$ and we can pull that factor out of the limit by the limit laws. Before we try L'Hôpital's rule again on the remaining factor, we should simplify it with algebra.

$$\frac{-2x^{-2}}{(1+3x)^{-2}} = \frac{2(1+3x)^2}{x^2}.$$

Now we can use L'Hôpital's rule, or even better, do a little more algebra and calculate the limit directly:

$$\lim_{x\to\infty}\frac{2(1+3x)^2}{x^2} = \lim_{x\to\infty} 2\left(\frac{1}{x}+3\right)^2 = 2(0+3)^2 = 18.$$

Altogether,

$$\lim_{x\to\infty}(1+3x)\sin(2/x) = \lim_{x\to\infty}\cos(2/x)\cdot\lim_{x\to\infty}\frac{-2x^{-2}}{-(1+3x)^{-2}} = 1\cdot 18 = 18.$$

2. Let's evaluate the indefinite integral first. There are two ways to proceed: by substitution or by the same kind of analysis that was used in the solution to Problem Set 3. By substitution, let $u = \sin x$. Then $du = \cos x\, dx$ and the indefinite integral becomes

$$\int \frac{\cos x}{\sqrt{1-\sin^2 x}}\,dx = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1}\sin x + C = x + C.$$

However, it is actually better (as we saw in the problem set) to think about this a little more. Note that, by the Pythagorean identity for trig functions,

$$\sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = |\cos x|,$$

so

$$\frac{\cos x}{\sqrt{1-\sin^2 x}} = \frac{\cos x}{|\cos x|},$$

i.e., sometimes the integrand is $+1$ and sometimes it is -1 (and sometimes it doesn't exist). On the interval $[-\pi/4, \pi/4]$ we have $\cos x > 0$ so $\cos x/|\cos x| = +1$ and the definite integral is

$$\int_{-\pi/4}^{\pi/4} 1\,dx = x\Big|_{-\pi/4}^{\pi/4} = \frac{\pi}{4} - -\frac{\pi}{4} = \frac{\pi}{2} = 1.5707$$

to four decimal places.