UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS
Mathematics 111
Final Examination
200610

Time: 3 hours
Instructors: Dr. D. Ruoff (-001)
Dr. C-H. Guo (-002)
Dr. A. Herman (-003) & (-004)
Dr. F. Labropulu (-L01) & (-L02)

NAME: _____

STUDENT NO.:_____

INSTRUCTIONS: Write all solutions on the test pages. Use the back of the page if necessary.

[10 marks] 1. Let $f(x) = \dfrac{e^x + 1}{x + 1}$.

(a) Find the equation of the tangent line to the graph of this function when $x = 0$.

(b) Find $(f^{-1})'(y)$ for $y = 2$.

[10 marks] 2. Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2\cos x}{6x^2}$

(b) $\lim_{x \to 0^+} x(\log_2 x)^2$

[5 marks] 3. Differentiate $y = (x^2 + 1)^{(3^x)}$.

[30 marks] 4. Evaluate the following integrals.

(a) $\int_0^1 5^{-2x}\, dx$

(b) $\int_0^{\frac{1}{2\sqrt{5}}} \frac{dt}{\sqrt{1-5t^2}}$

(c) $\int (\ln x)^2\, dx$

(d) $\int xe^{2x}\, dx$

(e) $\int \tan^{-1} x \, dx$

(f) $\int \tan^3 x \sec^3 x \, dx$

(g) $\int \frac{1}{x^2\sqrt{x^2+1}} \, dx$

(h) $\displaystyle\int_3^7 \frac{1}{\sqrt{x-3}}\,dx$

(i) $\displaystyle\int_0^\infty \frac{x}{(1+x^2)^3}\,dx$

(j) $\displaystyle\int \frac{5x^2-10x-8}{x^3+4x}\,dx$

[5 marks] 5. Solve the differential equation:

$$y\frac{dy}{dx} = e^{2x-y^2}.$$

[6 marks] 6. Solve the differential equation:

$$x^2\frac{dy}{dx} + 3xy = \frac{1}{x}, \quad y(1) = -1.$$

[6 marks] 7. Solve the differential equation:

$$xy\frac{dy}{dx} = 4x^2 + y^2.$$

[8 marks] 8. (a) Show that the differential equation

$$\cos(x+y)dx + (2y + \cos(x+y))dy = 0$$

is exact.

(b) Solve the differential equation given in part (a).

[10 marks] 9. Find the volume of the solid of revolution obtained when the region bounded by the curves $y = x^2$ and $y = 2x$ is rotated about the line $y = 5$.

[10 marks] 10. Find the area of the surface obtained when the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ $(1 \leq x \leq 3)$ is rotated about the x-axis.