

UNIVERSITY OF REGINA  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
MATH 122-001 Final Examination 200610

Time: 3 hours

Name: \_\_\_\_\_

Instructor: Dr. Edward Doolittle

Student #: \_\_\_\_\_

(marks) Show all of your work on the pages of this examination paper. Use the backs of the pages if necessary. Do not hand in any scratch paper. A non-programmable calculator of the type listed in the course outline is permitted; other non-programmable calculators are permitted with prior permission.

(10) 1. Consider the linear system

$$\begin{array}{rcl} 2x_2 - 8x_3 + 2x_4 & = & 4 \\ -x_1 + x_2 - 7x_3 + 3x_4 - 2x_5 & = & -1 \\ -x_1 & - & 3x_3 + 2x_4 + x_5 = -3 \\ 2x_1 - 3x_2 + 18x_3 - 7x_4 & = & 0 \end{array}$$

- (a) Solve the system.
- (b) Write the solution in parametric form.

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(10) 2. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}.$$

- (a) Show that  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$  are linearly independent.
- (b) Express  $\mathbf{u}$  as a linear combination of  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ .
- (c) What is  $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ ? Justify your answer.

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- (8) 3. Consider the linear transformation  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  given by

$$T(x_1, x_2, x_3, x_4) = (x_2 + 3x_3 - x_4, 2x_1 + x_2 - 4x_3 + 2x_4, 2x_1 + 3x_2 + 2x_3).$$

- (a) Find the standard matrix  $A$  of  $T$ .
- (b) Is  $T$  onto? Justify your answer.
- (c) Is  $T$  one-to-one? Justify your answer.

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- (12) 4. For each of the following matrices, either find the inverse matrix or explain why the inverse does not exist.

(a)  $\begin{bmatrix} 6 & 7 \\ 7 & 8 \end{bmatrix}$

(b)  $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

(c)  $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(d)  $\begin{bmatrix} 4 & 3 & -1 & 0 & 2 \\ 2 & 4 & 0 & 0 & 5 \\ -1 & 3 & 2 & 0 & 1 \\ 3 & 6 & 3 & 0 & -2 \\ 0 & 5 & -4 & 0 & 6 \end{bmatrix}$

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- (8) 5. Let  $A$  and  $B$  be two  $4 \times 4$  matrices with  $\det(A) = 3$  and  $\det(B) = -7$ .
- (a) What is  $\det(B^{-2}AB^2)$ ?
  - (b) What is  $\det(2A)$ ?
  - (c) What is  $\det(3A^{-1})$ ?
  - (d) What is  $\det((3A)^{-1})$ ?

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- (12) 6. Find the following.
- (a) The area of the quadrilateral in  $\mathbb{R}^2$  with vertices  $(-1, 0)$ ,  $(0, 5)$ ,  $(1, -4)$ , and  $(2, 1)$ .
  - (b) The area of the triangle in  $\mathbb{R}^3$  with vertices  $(2, 2, 0)$ ,  $(-1, 0, 2)$ ,  $(0, 4, 3)$ .
  - (c) The volume of the parallelepiped in  $\mathbb{R}^3$  with one vertex at the origin and adjacent vertices at  $(1, 0, -2)$ ,  $(1, 2, 4)$ , and  $(7, 1, 0)$ .

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(10) 7. Consider the system

$$\begin{aligned}6sx_1 + 4x_2 &= 5 \\ 3x_1 + 2sx_2 &= -2\end{aligned}$$

- (a) Determine the values of the parameter  $s$  for which the system has a unique solution.
- (b) For such values of  $s$ , determine  $x_1$  and  $x_2$  in terms of  $s$ .

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(12) 8. Consider the matrix

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}.$$

- (a) Find the characteristic polynomial of  $A$ .
- (b) Find the eigenvalues of  $A$ .
- (c) For each eigenvalue, find a basis for the corresponding eigenspace.
- (d) What are the eigenvalues of  $A^2$ ?

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(10) 9. Let

$$\mathbf{a} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}.$$

- (a) Find  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ .
- (b) Find  $\mathbf{u}_1 = \text{proj}_{\mathbf{c}} \mathbf{u}$ .
- (c) Find  $\mathbf{u}_2 = \mathbf{u} - \mathbf{u}_1$ .
- (d) Show that  $\mathbf{u}_1$  is orthogonal to  $\mathbf{a}$  and to  $\mathbf{b}$ .
- (e) Show that  $\mathbf{u}_2$  is a linear combination of  $\mathbf{a}$  and  $\mathbf{b}$ .

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- (8) 10. (a) Suppose that the columns of the  $n \times n$  matrix  $A$  are a basis for  $\mathbb{R}^n$ . Show that the columns of  $A^2$  are also a basis for  $\mathbb{R}^n$ .
- (b) Show that  $\lambda$  is an eigenvalue of  $A$  if and only if  $\lambda$  is an eigenvalue of  $A^T$ .