

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 122 200610 Midterm Test 1 (A Version)

Time: 50 minutes

Instructor: Dr. Edward Doolittle

Name: _____

Student #: _____

(marks) Please do all questions. You have 50 minutes to do the exam, which is worth 50 marks; you should try to earn one mark per minute. A non-programmable calculator is allowed but is not necessary. You may leave early if you can do so without disturbing any of your colleagues. If you finish early, I suggest you check your work thoroughly.

(20) 1. Consider the vectors

$$\mathbf{u}_1 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 1 \\ -11 \\ -2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 9 \\ -11 \\ -7 \end{bmatrix}.$$

(a) Is \mathbf{b} a linear combination of \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 ? Justify your answer.

(b) Describe geometrically the set of all x_1, x_2, x_3 such that $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 = \mathbf{b}$.

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(c) Are \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 linearly independent? Justify your answer.

(d) Do \mathbf{u}_1 , \mathbf{u}_2 , and \mathbf{u}_3 span \mathbb{R}^3 ? Justify your answer.

(10) 2. Consider the mapping $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$T(x_1, x_2, x_3) = (2x_1 - x_2, x_2 + x_3, x_3 + x_1).$$

(a) Show that T is a linear transformation.

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(b) Find the standard matrix of T .

(15) 3. Consider the matrix transformation $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 3 & -10 & -6 \\ -2 & -5 & 16 & 11 \\ 2 & 3 & k & -9 \end{bmatrix};$$

k is a parameter.

(a) What is the domain of T ? The codomain of T ?

(b) For what value(s) of k is T onto? Justify your answer.

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(c) For what value(s) of k is T one-to-one? Justify your answer.

(5) 4. Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation for which

$$T(4\mathbf{e}_1 - 5\mathbf{e}_2) = 3\mathbf{e}_1 - 2\mathbf{e}_2$$

$$T(-3\mathbf{e}_1 + 4\mathbf{e}_2) = -\mathbf{e}_1 + \mathbf{e}_2.$$

Find the standard matrix of T .