UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 122 200610 Midterm Test 1 (B Version)

Time: 50 minutes
Instructor: Dr. Edward Doolittle

Name: _____
Student #: _____

(marks)

Please do all questions. You have 50 minutes to do the exam, which is worth 50 marks; you should try to earn one mark per minute. A non-programmable calculator is allowed but is not necessary. You may leave early if you can do so without disturbing any of your colleagues. If you finish early, I suggest you check your work thoroughly.

(20) 1. Consider the vectors

$$\mathbf{u}_1 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} -3 \\ 3 \\ -6 \end{bmatrix}, \ \mathbf{u}_3 = \begin{bmatrix} 8 \\ -10 \\ 22 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 9 \\ -6 \\ 9 \end{bmatrix}.$$

(a) Is $\mathbf{b}$ a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$? Justify your answer.

(b) Describe geometrically the set of all x_1, x_2, x_3 such that $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 = \mathbf{b}$.

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(c) Are $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ linearly independent? Justify your answer.

(d) Do $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ span $\mathbb{R}^3$? Justify your answer.

(10) 2. Consider the mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x_1, x_2, x_3) = (3x_1 - x_2, 2x_2 + x_3, x_3 - x_1).$$

(a) Show that T is a linear transformation.

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(b) Find the standard matrix of T.

(15) 3. Consider the matrix transformation $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & -2 & 7 & 11 \\ 2 & -3 & 12 & 19 \\ -3 & 0 & k & -15 \end{bmatrix};$$

k is a parameter.

(a) What is the domain of T? The codomain of T?

(b) For what value(s) of k is T onto? Justify your answer.

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(c) For what value(s) of k is T one-to-one? Justify your answer.

(5) 4. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation for which

$$T(5\mathbf{e}_1 - 3\mathbf{e}_2) = -\mathbf{e}_1 - 2\mathbf{e}_2$$
$$T(-8\mathbf{e}_1 + 5\mathbf{e}_2) = -2\mathbf{e}_1 + 3\mathbf{e}_2.$$

Find the standard matrix of T.