

UNIVERSITY OF REGINA  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
MATH 122 200610 Midterm Test 2 (A Version)

Time: 50 minutes

Instructor: Dr. Edward Doolittle

Name: \_\_\_\_\_

Student #: \_\_\_\_\_

(marks) Please do all questions. You have 50 minutes to do the exam, which is worth 50 marks; you should try to earn one mark per minute. A non-programmable calculator is allowed but is not necessary. You may leave early if you can do so without disturbing any of your colleagues. If you finish early, I suggest you check your work thoroughly.

1. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 6 \\ 1 & 5 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(5) (a) Compute  $AD$  and  $DA$ .

(5) (b) Express  $D$  as a product of elementary matrices. (Hint: consider what row operations would have the same effect as the matrix product  $DA$ .)

2. Let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}.$$

(5) (a) Find the inverse of  $A$ .

(5) (b) Is it possible for the equation  $A\mathbf{x} = \mathbf{b}$  to have no solutions for some  $\mathbf{b} \in \mathbb{R}^3$ ? To have more than one solution for some  $\mathbf{b} \in \mathbb{R}^3$ ? Is the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by  $T(\mathbf{x}) = A\mathbf{x}$  onto? One-to-one? Justify your answers.

(5) (c) Use  $A^{-1}$  to solve the equation  $A\mathbf{x} = \mathbf{u}$ , and use matrix multiplication to check your answer.

3. Let

$$B = \begin{bmatrix} 1 & -4 \\ 4 & 2 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(5) (a) Calculate  $(B - 3I)B + 18I$ .(5) (b) Using the above, or otherwise, find  $B^{-1}$ .

4. Let

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix}.$$

(5) (a) Find a basis for the null space of  $A$ .

- (5) (b) Find a basis for the column space of  $A$ , and express the non-basis columns of  $A$  as linear combinations of the basis vectors.

- (5) 5. Suppose  $A$ ,  $B$ , and  $X$  are  $n \times n$  matrices with  $A$ ,  $X$ , and  $A - AX$  invertible, and suppose  $(A - AX)^{-1} = X^{-1}B$ . Solve the equation for  $X$  in terms of  $A$  and  $B$ . If you need to invert a matrix, explain why that matrix is invertible.