

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 122 200610 Midterm Test 2 (C Version)

Time: 50 minutes

Instructor: Dr. Edward Doolittle

Name: _____

Student #: _____

(marks) Please do all questions. You have 50 minutes to do the exam, which is worth 50 marks; you should try to earn one mark per minute. A non-programmable calculator is allowed but is not necessary. You may leave early if you can do so without disturbing any of your colleagues. If you finish early, I suggest you check your work thoroughly.

1. Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 1 & 5 & 7 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(5) (a) Compute AD and DA .

(5) (b) Express D as a product of elementary matrices. (Hint: consider what row operations would have the same effect as the matrix product DA .)

2. Let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 9 \\ 6 \end{bmatrix}.$$

(5) (a) Find the inverse of A .

(5) (b) Is it possible for the equation $A\mathbf{x} = \mathbf{b}$ to have no solutions for some $\mathbf{b} \in \mathbb{R}^3$? To have more than one solution for some $\mathbf{b} \in \mathbb{R}^3$? Is the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(\mathbf{x}) = A\mathbf{x}$ onto? One-to-one? Justify your answers.

(5) (c) Use A^{-1} to solve the equation $A\mathbf{x} = \mathbf{u}$, and use matrix multiplication to check your answer.

3. Let

$$B = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(5) (a) Calculate $(B - 3I)B + 2I$.(5) (b) Using the above, or otherwise, find B^{-1} .

4. Let

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -2 & 7 \\ -2 & -4 & 5 & 0 & -6 \end{bmatrix}.$$

(5) (a) Find a basis for the null space of A .

- (5) (b) Find a basis for the column space of A , and express the non-basis columns of A as linear combinations of the basis vectors.

- (5) 5. Suppose A , B , and X are $n \times n$ matrices with A , X , and $A - AX$ invertible, and suppose $(A - AX)^{-1} = X^{-1}B$. Solve the equation for X in terms of A and B . If you need to invert a matrix, explain why that matrix is invertible.