

MATH122 200610 Problem Set 3

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The following problems from the exercises in sections 1.4 and 1.5 may appear on the quiz on February 1.

1. (Based on 1.4.12, 1.4.14) For each of the following, determine whether the vector \mathbf{b} is in the subset of \mathbb{R}^3 spanned by the columns of A :

$$(a) A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \qquad (b) A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$$

2. (Based on 1.4.16) For each of the following, show that it is not the case that the equation $A\mathbf{x} = \mathbf{b}$ has a solution for all possible \mathbf{b} by giving an example of a vector \mathbf{b} for which the equation does not have a solution. Describe the set of all \mathbf{b} for which $A\mathbf{x} = \mathbf{b}$ *does* have a solution.

$$(a) A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (b) A = \begin{bmatrix} 0 & 5 & -2 \\ 4 & 8 & 3 \\ -4 & 2 & -7 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

3. (Based on 1.4.38 and 1.4.40) Determine whether the columns of each of the following matrices span \mathbb{R}^4 .

$$(a) \left\{ \begin{bmatrix} 5 & -7 & -4 & 9 \\ 6 & -8 & -7 & 5 \\ 4 & -4 & -9 & -9 \\ -9 & 11 & 16 & 7 \end{bmatrix} \right\} \qquad (b) \left\{ \begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \right\}$$

4. (Based on 1.5.6 and 1.5.16) For each of the following, describe all solutions to the equation $A\mathbf{x} = \mathbf{b}$ in parametric form.

$$(a) A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (b) A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 7 \\ -6 \end{bmatrix}$$

5. (Based on 1.5.18) Describe and compare the solution sets of the two equations $x_1 - 3x_2 + 5x_3 = 0$ and $x_1 - 3x_2 + 5x_3 = 4$.

Other problems which will help you learn the material can be found in section 1.4, practice problems 1–2 and exercises 1–19, 21–22, 25–28, 37–40 (try the odd numbers first), and in section 1.5, practice problems 1–2 and exercises 1–21 (again, try the odd numbers first). Students who would like obtain an A in the course should also try exercises 1.4.29–36, 1.4.41–42 and 1.5.26–40.