MATH122 200610 Problem Set 5

Edward Doolittle

Wednesday, February 8, 2006

1. (Based on 1.8.4 and 1.8.6.) Define the linear transformation T by $T(\mathbf{x}) = A\mathbf{x}$. In each of the following cases, find a vector $\mathbf{x}$ the image of which under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

(a) $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \\ -3 & 5 & -4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 9 \\ 3 \\ -6 \end{bmatrix}$

2. (Based on 1.8.10, 1.8.12, and 1.8.38.) Define the linear transformation T by $T(\mathbf{x}) = A\mathbf{x}$. In each of the following cases, find all $\mathbf{x} \in \mathbb{R}^4$ that are mapped to the zero vector $\mathbf{0}$ by T. Decide whether the given $\mathbf{b}$ is in the range of T, and justify your decision.

(a) $A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} -9 & -4 & -9 & 4 \\ 5 & -8 & -7 & 6 \\ 7 & 11 & 16 & -9 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

3. (Based on 1.9.4, 1.9.6, and 1.9.8.) Find the standard matrix of each of the following geometric transformations. You may assume that the transformation is linear.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points about the origin through an angle of $-\pi/4$ radians.

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a horizontal shear transformation that leaves $\mathbf{e}_1$ unchanged and maps $\mathbf{e}_2$ to $\mathbf{e}_2 + 3\mathbf{e}_1$.

(c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the vertical x_2-axis and then reflects points through the line $x_2 = x_1$.

4. (Based on 1.9.16, 1.9.18, and 1.9.20.) For each of the following transformations T, find the domain and codomain, show that the mapping is linear, and find the standard matrix of the mapping.

(a) $T(x_1, x_2) = (x_1 - x_2, -2x_1 + x_2, x_1)$

(b) $T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2)$

(c) $T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4$.

5. For each of the linear transformations in the previous question, determine whether the transformation is (a) 1–1, (b) onto.

Other problems which will help you learn the material can be found in section 1.8, practice problems 1–2 and exercises 1–20 (try the odd numbers first), and in section 1.9, practice problem and exercises 1–22 and 25–28 (again, try the odd numbers first). Students who would like obtain an A in the course should also try exercises 1.8.23–36 and 1.9.29–36.