

MATH122 200610 Problem Set 6

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The following problems from chapter 2.1 may appear on the quiz on March 1.

1. (Based on 2.1.2.) Let

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}.$$

Compute the following expressions if they are defined. If an expression is undefined, explain why.

(a) $A + 2B$ (b) $3C - E$ (c) CB (d) EB

2. (Based on 2.1.8.) How many rows does B have if BC is a 3×4 matrix? How many columns does C have?

3. (Based on 2.1.10.) Let

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}, B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}, C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}.$$

Verify that $AB = AC$ and yet $B \neq C$. (This shows that cancellation of the matrix A in the equation is not possible; one has to be careful about cancellation in matrix equations in general. We will learn more about it in chapter 2.2.)

4. (Based on 2.1.12.) Let

$$A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}.$$

Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B . (This shows that another rule of ordinary algebra, namely that $xy = 0$ implies $x = 0$ or $y = 0$, is not true for matrix algebra.)

5. (Based on 2.1.22.) Show that if the columns of B are linearly dependent, then so are the columns of AB . (Hint: The columns of B are linearly dependent if and only if a certain matrix-vector equation is satisfied.)

6. (Based on 2.1.28.) If \mathbf{u} and \mathbf{v} are in \mathbb{R}^n , are $\mathbf{u}^T \mathbf{v}$ and $\mathbf{v}^T \mathbf{u}$ defined? If so, what are their sizes, and how are they related? Similarly, are $\mathbf{u} \mathbf{v}^T$ and $\mathbf{v} \mathbf{u}^T$ defined? If so, what are their sizes, and how are they related?

Other problems which will help you learn the material can be found in section 2.1, practice problems 1–2 and exercises 1–13 (try the odd numbers first). Students who would like obtain an A in the course should also try exercises 2.1.17–34.