MATH122 200610 Problem Set 7

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The following problems from chapters 2.2 and 2.3 may appear on the test on March 15.

1. (Based on 2.2.2, 2.2.4, and 2.2.6.) For each of the following systems, use Cramer's rule (Theorem 4) to find the inverse of the coefficient matrix, and then use the inverse matrix to solve the system.

   (a) $\begin{aligned} 3x_1 + 2x_2 &= -1 \\ 7x_1 + 4x_2 &= 5 \end{aligned}$ (b) $\begin{aligned} 3x_1 - 4x_2 &= -3 \\ 7x_1 - 8x_2 &= -4 \end{aligned}$ (c) $\begin{aligned} 8x_1 + 5x_2 &= -9 \\ -7x_1 - 5x_2 &= 11 \end{aligned}$

2. (Based on 2.2.30, 2.2.32, and 2.2.34.) Find the inverses (if they exist) of the following matrices using row reduction.

   (a) $\begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

3. (Based on 2.3.4, 2.3.6, and 2.3.8.) Determine which of the following matrices are invertible. Use as few calculations as possible to make your determination. For those that are invertible, find the inverse.

   (a) $\begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 5 & 9 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

4. (Based on 2.3.34.) Show that each of the following linear transformations $T : \mathbb{R}^n \to \mathbb{R}^n$ is invertible, and find a formula for T^{-1}.

   (a) $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$
   (b) $T(x_1, x_2, x_3) = (x_1, x_1 - 2x_2, x_1 - 2x_2 + 3x_3)$
   (c) $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, -3x_1 - x_2 + 2x_3, 5x_2 + 3x_3)$

5. (Based on 2.3.16, 2.3.18, 2.3.22, 2.3.24, 2.3.26, 2.3.28.) Use Theorem 8 to answer each of the following.

   (a) Can a 5×5 matrix be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?
   (b) If C is a 6×6 matrix and the equation $C\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^6$, is it possible that for some $\mathbf{b}$, the equation $C\mathbf{x} = \mathbf{b}$ has more than one solution? Why or why not?
   (c) If G is an $n \times n$ matrix and the equation $G\mathbf{x} = \mathbf{y}$ has more than one solution for some $\mathbf{y}$ in $\mathbb{R}^n$, can the columns of G span $\mathbb{R}^n$? Why or why not?
   (d) If L is an $n \times n$ matrix and the equation $L\mathbf{x} = \mathbf{0}$ has only the trivial solution, do the columns of L span $\mathbb{R}^n$? Why or why not?
   (e) Explain why the columns of A^2 span $\mathbb{R}^n$ if the columns of A are linearly independent.
   (f) Show that if AB is invertible, then so is B.

Other problems which will help you learn the material can be found in section 2.2, practice problems 1–2 and exercises 1–8 and 29–35; and section 2.3, practice problems 1–3 and exercises 1–8 and 13–37. Students who would like obtain an A in the course should also try exercises 2.2.11–28, 2.2.37–38, and 2.3.38–40.