MATH122 200610 Problem Set 8 DRAFT

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The following problems from chapters 2.8 and 2.9 may appear on the test on March 15.

1. (Based on 2.8.6.) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ -7 \\ 9 \\ 7 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ -8 \\ 6 \\ 5 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} -4 \\ 10 \\ -7 \\ -5 \end{bmatrix}.$$

   Determine whether $\mathbf{u}$ is in the subspace of $\mathbb{R}^4$ generated by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

2. (Based on 2.8.24 and 2.8.26.) Find a basis for $\operatorname{Col} A$ and $\operatorname{Nul} A$ for each of the following matrices.

   (a) $\begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 5 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{bmatrix}$

3. (Based on 2.9.4 and 2.9.6.) For each of the following, assume that the vector $\mathbf{x}$ is in a subspace H of $\mathbb{R}^n$ with basis $B = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the B-coordinate vector of $\mathbf{x}$.

   (a) $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$ (b) $\mathbf{b}_1 = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$

4. (Based on 2.9.14.) Find a basis for the subspace spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \\ 5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ -3 \\ -1 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 2 \\ -6 \\ 8 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} -1 \\ 4 \\ -7 \\ 7 \end{bmatrix}, \mathbf{v}_5 = \begin{bmatrix} 3 \\ -8 \\ 9 \\ -5 \end{bmatrix}.$$

   What is the dimension of the subspace?

5. (Based on 2.9.20, 2.9.22, and 2.9.24.)
   (a) What is the rank of a 4×5 matrix the null space of which has dimension 3?
   (b) Show that a set of five vectors is linearly dependent if it spans a subspace of dimension 4 in $\mathbb{R}^n$.
   (c) Construct a 4×3 matrix with rank 1.

Other problems which will help you learn the material can be found in section 2.8, practice problems 1–3 and exercises 1–20 and 23–26; and section 2.9, practice problems 1–3 and exercises 1–16 and 19–26. Students who would like obtain an A in the course should also try exercises 2.8.27–36, and 2.9.27–28.