

# MATH122 200610 Problem Set 9

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The following problems from chapters 3.1 and 3.2 may appear on the quiz on March 29.

1. (Based on 3.1.4, 3.1.10, and 3.1.14.) Compute the following determinants using cofactor expansions.

$$(a) \begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} \quad (b) \begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} \quad (c) \begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix}$$

2. (Based on 3.1.22 and 3.1.24.) Explore the effect of an elementary row operation on the determinant of a matrix. In each case, find the determinants, state the row operation and describe how it affects the determinant.

$$(a) \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a & b \\ kc & kd \end{bmatrix} \quad (b) \begin{bmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{bmatrix}$$

3. (Based on 3.2.8, 3.2.12, and 3.2.14.) Find the following determinants by row reduction to row echelon form and/or cofactor expansions.

$$(a) \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{vmatrix} \quad (b) \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix} \quad (c) \begin{vmatrix} -3 & -2 & 1 & -4 \\ 1 & 3 & 0 & -3 \\ -3 & 4 & -2 & 8 \\ 3 & -4 & 0 & 4 \end{vmatrix}$$

4. (Based on 3.2.22, 3.3.24, and 3.2.26.) Use determinants to decide whether each of the following sets of vectors is linearly independent or not.

$$(a) \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix} \quad (b) \begin{bmatrix} 4 \\ 6 \\ -7 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 6 \end{bmatrix} \quad (c) \begin{bmatrix} 3 \\ 5 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}$$

5. (Based on 3.2.40.) Let  $A$  and  $B$  be  $4 \times 4$  matrices with  $\det A = -1$  and  $\det B = 2$ . Compute

$$(a) \det AB \quad (b) \det B^5 \quad (c) \det 2A \quad (d) \det A^T A \quad (e) \det B^{-1}AB$$

Other problems which will help you learn the material can be found in section 3.1, practice problem and exercises 1–38; and section 2.9, practice problems 1–2 and exercises 1–26, 29–30, and 37–40. Students who would like obtain an A in the course should also try exercises 3.1.41–46, 3.2.31–36, and 3.2.41–44.