MATH122 200610 Problem Set 10

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Questions based on the following problems from Lay chapter 3.3 and Stewart chapters 13.3 and 13.4 (referred to as S13.3 and S13.4 below) may appear on the final exam. Most of the geometry covered in the course appears in this problem set. Note that this problem set spans two pages, unlike the previous problem sets.

1. (Based on 3.3.2, 3.3.4 and 3.3.6.) Use Cramer's rule to find the solutions to the following systems.

   (a) $\begin{aligned} 4x_1 + x_2 &= 6 \\ 5x_1 + 2x_2 &= 7 \end{aligned}$ (b) $\begin{aligned} -5x_1 + 3x_2 &= 9 \\ 3x_1 - x_2 &= -5 \end{aligned}$ (c) $\begin{aligned} 2x_1 + x_2 + x_3 &= 4 \\ -x_1 + 2x_3 &= 2 \\ 3x_1 + x_2 + 3x_3 &= -2 \end{aligned}$

2. (Based on 3.3.8 and 3.3.10.) For each of the following systems, determine the values of the parameter s for which the system has a unique solution, and find the solution.

   (a) $\begin{aligned} 3sx_1 - 5x_2 &= 3 \\ 9x_1 + 5sx_2 &= 2 \end{aligned}$ (b) $\begin{aligned} 2sx_1 + x_2 &= 1 \\ 3sx_1 + 6sx_2 &= 2 \end{aligned}$ (c) $\begin{aligned} 2x_1 + sx_2 + 3x_3 &= 14 \\ x_1 - x_2 + x_3 &= 0 \\ x_1 + 4x_2 - 2x_3 &= -28 \end{aligned}$

3. (Based on 3.3.12, 3.3.14, and 3.3.16.) Use Cramer's rule to find the inverse of each of the following matrices.

   (a) $\begin{bmatrix} 1 & 1 & 3 \\ 2 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

4. (Based on 3.3.20, 3.3.22, and 3.3.24.) Find the area or volume of the following geometrical figures.

   (a) The parallelogram with one vertex at the origin and adjacent vertices at $(-1, 3)$ and $(4, -5)$.

   (b) The parallelogram with vertices $(0, -2)$, $(6, -1)$, $(-3, 1)$, and $(3, 2)$. You should check that the figure really is a parallelogram.

   (c) The parallelepiped with one vertex at the origin and adjacent vertices at $(1, 4, 0)$, $(-2, -5, 2)$, and $(-1, 2, -1)$.

5. (Based on S13.3.16, S13.3.18, and S13.3.20.) Find the angle between the given vectors. (You can measure angles in either degrees or radians, provided that you indicate the units. First find an exact expression and then an approximation to the nearest degree or hundredth of a radian.)

   (a) $\mathbf{a} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ (c) $\begin{aligned} \mathbf{a} &= 2\mathbf{i} - \mathbf{j} + \mathbf{k} \\ \mathbf{b} &= 3\mathbf{i} + 2\mathbf{j} - \mathbf{k} \end{aligned}$

6. (Based on S13.3.24.) Determine whether the given vectors are orthogonal, parallel, or neither.

(a) $\begin{bmatrix} -3 \\ 9 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ -12 \\ -8 \end{bmatrix}$ (b) $\begin{array}{l} \mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} \\ \mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} \end{array}$ (c) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

7. (Based on S13.3.30, S13.3.32, and S13.3.33.) Find the direction cosines and direction angles of the following vectors.

(a) $\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$ (b) $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ (c) $\begin{bmatrix} -c \\ c \\ -c \end{bmatrix}, c > 0$

8. (Based on S13.3.36, S13.3.38, and S13.3.40.) Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

(a) $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ (b) $\mathbf{a} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$ (c) $\begin{array}{l} \mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k} \\ \mathbf{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k} \end{array}$

9. (Based on S13.4.2 and S13.4.4.) Find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

(a) $\mathbf{a} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ (b) $\begin{array}{l} \mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k} \\ \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k} \end{array}$

10. (Based on S13.4.16.) Find two unit vectors orthogonal to both $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $2\mathbf{i} + \mathbf{k}$.

11. (Based on S13.4.24, S13.4.30, and S13.4.32.) Use the cross product or the triple product to find the area or volume of the following geometrical figures.

    (a) The parallelogram with vertices $A(-2, 1)$, $B(0, 4)$, $C(4, 2)$, and $D(2, -1)$. (You should check that the figure really is a parallelogram.)

    (b) The parallelpiped determined by the vectors $\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{i} - \mathbf{j} + \mathbf{k}$, $-\mathbf{i} + \mathbf{j} + \mathbf{k}$.

    (c) The parallelepiped with adjacent edges PQ, PR, and PS where the points P, Q, R, and S are $(0, 1, 2)$, $(2, 4, 5)$, $(-1, 0, 1)$, and $(6, -1, 4)$ respectively.

12. (Based on S13.4.26 and S13.4.28.) Find a vector orthogonal to the plane through the points P, Q, and R, and find the area of triangle PQR.

(a) $P(2, 1, 5)$, $Q(-1, 3, 4)$, $R(3, 0, 6)$ (b) $P(2, 0, -3)$, $Q(3, 1, 0)$, $R(5, 2, 2)$

Other problems which will help you learn the material can be found in section 3.3, practice problem and exercises 1–24 and 27–28; S13.3, exercises 1–24, 29–33, and 35–40; and S13.4, exercises 1–5, 8–9, 13–16, and 23–32. Students who would like obtain an A in the course should also try exercises 3.3.25–26, 3.3.29–30, 3.3.32, S13.3.25–28, S13.3.34, S13.3.41–44, S13.3.49–59, S13.3.17–22, S13.3.33–34, and S13.3.38–46.