

# MATH122 200610 Problem Set 11

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1. (Based on 5.1.4 and 5.1.6.) For each of the following matrices, determine whether the given vector is an eigenvector for the matrix. If it is an eigenvector, find the corresponding eigenvalue.

(a)  $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}; \begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix}$       (b)  $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}; \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$       (c)  $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 5 \end{bmatrix}; \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

2. (Based on 5.1.2 and 5.1.8.) For each of the following matrices, determine whether the given number is an eigenvalue for the matrix. If it is an eigenvalue, find one corresponding eigenvector.

(a)  $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}; \lambda = -2$       (b)  $\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}; \lambda = 3$       (c)  $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}; \lambda = 4$

3. (Based on 5.1.10, 5.1.14, and 5.1.16.) For each of the following matrices, find a basis for the eigenspace corresponding to the given eigenvalue.

(a)  $\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}; 4$       (b)  $\begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}; -2$       (c)  $\begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}; 4$

4. (Based on 5.2.2, 5.2.6, and 5.2.8.) Find the characteristic polynomials and eigenvalues of the following.

(a)  $\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$       (b)  $\begin{bmatrix} 3 & -4 \\ 4 & 8 \end{bmatrix}$       (c)  $\begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$

5. (Based on 5.2.10, 5.2.12, and 5.2.14.) Find the characteristic polynomials of the following matrices.

(a)  $\begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$       (b)  $\begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$       (c)  $\begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$

6. Find the characteristic polynomial and the eigenvalues of each of the following matrices. For each eigenvalue, find a basis for the corresponding eigenspace.

(a)  $\begin{bmatrix} -1 & -2 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & -1 \end{bmatrix}$       (b)  $\begin{bmatrix} 5 & -3 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$       (c)  $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

If you're having trouble with (c), look elsewhere on the page for a hint.

Other problems which will help you learn the material can be found in section 5.1, practice problems 1 and 2 and exercises 1–20; and chapter 5.2, practice problem and exercises 1–18. Students who would like obtain an A in the course should also try exercises 5.1.23–30, 5.1.35–36, and 5.2.19–20.