# MATH122 200610 Quiz 2A Solutions

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1. The equation corresponds to the augmented matrix

$$\left[\begin{array}{ccc|c} -4 & -13 & -1 & b_1 \\ 4 & 8 & 3 & b_2 \\ -4 & 2 & -7 & b_3 \end{array}\right].$$

Adding 1 times row 1 to row 2, and $-1$ times row 1 to row 3,

$$\left[\begin{array}{ccc|c} -4 & -13 & -1 & b_1 \\ 0 & -5 & 2 & b_1 + b_2 \\ 0 & 15 & -6 & -b_1 + b_3 \end{array}\right].$$

Adding 3 times row 2 to row 3,

$$\left[\begin{array}{ccc|c} -4 & -13 & -1 & b_1 \\ 0 & -5 & 2 & b_1 + b_2 \\ 0 & 0 & 0 & 2b_1 + 3b_2 + b_3 \end{array}\right].$$

The system is now in row echelon form. It does not have a solution if and only if it has a row of the form $\left[\begin{array}{ccc|c} 0 & 0 & 0 & \Box \end{array}\right]$. That can be arranged, for example, by setting $b_1 = 1$, $b_2 = 0$, and $b_3 = 0$. The set of all $\mathbf{b}$ for which the system does have a solution is characterized by the equation $2b_1 + 3b_2 + b_3 = 0$.

2. The system is described by the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & -1 \\ -3 & -6 & 3 & 0 \\ 2 & 3 & -4 & 2 \end{array}\right].$$

Adding 3 times row 1 to row 2, and $-2$ times row 1 to row 3,

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & -1 \\ 0 & 3 & 6 & -3 \\ 0 & -3 & -6 & 4 \end{array}\right].$$

Adding 1 times row 2 to row 3,

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & -1 \\ 0 & 3 & 6 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

The system is now in row echelon form. Since it contains a row of the form $\left[\begin{array}{ccc|c} 0 & 0 & 0 & \Box \end{array}\right]$, there is no solution.

3. Conservation of $H$ gives the equation

$$x_1 + x_3 = 2x_6.$$

Conservation of $I$ gives

$$x_1 + 2x_2 = x_5,$$

conservation of $O$ gives

$$3x_1 = x_6,$$

conservation of $Fe$ gives

$$x_2 = x_4,$$

and conservation of $Cl$ gives

$$x_3 = 3x_4 + x_5.$$

The equations are captured by the augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 1 & 2 & 0 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \end{array}\right].$$

Adding $-1$ times row 1 to row 2 and $-3$ times row 1 to row 3,

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 2 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & -3 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \end{array}\right].$$

Swapping row 2 and row 4 gives

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 5 & 0 \\ 0 & 2 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \end{array}\right].$$

Adding $-2$ times row 2 to row 4 gives

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 5 & 0 \\ 0 & 0 & -1 & 2 & -1 & 2 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \end{array}\right].$$

Swapping row 3 and row 5 gives

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 2 & 0 \\ 0 & 0 & -3 & 0 & 0 & 5 & 0 \end{array}\right].$$

Adding 1 times row 3 to row 4 and 3 times row 3 to row 5 gives

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -2 & 2 & 0 \\ 0 & 0 & 0 & -9 & -3 & 5 & 0 \end{array}\right].$$

Adding $-9$ times row 4 to row 5 gives

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 15 & -13 & 0 \end{array}\right].$$

The system is in row echelon form. To put it into reduced row echelon form, multiply row 4 by $-1$ and row 5 by $1/15$ to obtain

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -13/15 & 0 \end{array}\right].$$

Add $-2$ times row 5 to row 4 and 1 times row 5 to row 3 to obtain

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & -13/15 & 0 \\ 0 & 0 & 0 & 1 & 0 & -4/15 & 0 \\ 0 & 0 & 0 & 0 & 1 & -13/15 & 0 \end{array}\right].$$

Add 3 times row 4 to row 3, and 1 times row 4 to row 2 to obtain

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -4/15 & 0 \\ 0 & 0 & 1 & 0 & 0 & -25/15 & 0 \\ 0 & 0 & 0 & 1 & 0 & -4/15 & 0 \\ 0 & 0 & 0 & 0 & 1 & -13/15 & 0 \end{array}\right].$$

Finally, add $-1$ times row 3 to row 1 to obtain

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -5/15 & 0 \\ 0 & 1 & 0 & 0 & 0 & -4/15 & 0 \\ 0 & 0 & 1 & 0 & 0 & -25/15 & 0 \\ 0 & 0 & 0 & 1 & 0 & -4/15 & 0 \\ 0 & 0 & 0 & 0 & 1 & -13/15 & 0 \end{array}\right].$$

The solution to the system is

$$\begin{aligned} x_1 &= \frac{1}{3}s \\ x_2 &= \frac{4}{15}s \\ x_3 &= \frac{5}{3}s \\ x_4 &= \frac{4}{15}s \\ x_5 &= \frac{13}{15}s \\ x_6 &= s. \end{aligned}$$

The smallest positive value of $s$ which gives an all integer solution is $s = 15$ which gives $x_1 = 5$, $x_2 = 4$, $x_3 = 25$, $x_4 = 4$, $x_5 = 13$, and $x_6 = 15$. You should check that those values balance the reaction.