MATH122 200610 Quiz 4A Solutions DRAFT

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1. To evaluate these expressions we use the formulas $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$. For the last question the identities $(XY)^{-1} = Y^{-1}X^{-1}$ (note the reversal of order) and $(B^T)^{-1} = (B^{-1})^T$ may also come in handy, depending on how you solve the problem.
   (a) $\det(AB) = \det(A)\det(B) = 4(-3) = -12$.
   (b) $\det(3B) = \det((3I)B) = \det(3I)\det(B) = 3^3(-3) = -81$.
   (c) $\det(A^3) = (\det(A))^3 = 4^3 = 64$.
   (d) $\det(A^TB^{-1}) = \det(A^T)(\det(B))^{-1} = 4/(-3) = -4/3$.
   (e) $\det((B^TB)^{-1}B^T) = \det(B^{-1}(B^T)^{-1}B^T) = \det(B^{-1}) = (\det(B))^{-1} = -1/3$.
2. By the invertible matrix theorem, the given vectors are linearly independent if and only if the determinant

$$\Delta = \begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$$

is non-zero. There are many ways to evaluate the determinant. Since there is no obvious column or row full of zeros in which to expand the determinant, we try row reduction. Adding 2 times row 1 to row 2, -3 times row 1 to row 3, and -1 times row 1 to row 4 gives

$$\Delta = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{vmatrix}.$$

We could continue with the row reduction, but if we notice that the third and fourth rows are identical, we could simply add -1 times row 3 to row 4 to obtain

$$\Delta = \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{vmatrix}.$$

Now expanding in the fourth row gives $\Delta = 0$ and we can conclude that the given vectors are linearly dependent.