MATH122 200610 Quiz 4B Solutions DRAFT

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Wednesday, March 29, 2006

1. To evaluate these expressions we use the formulas $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$. For the last question the identities $(AB)^{-1} = B^{-1}A^{-1}$ (note the reversal of order) and $(B^T)^{-1} = (B^{-1})^T$ may also come in handy, depending on how you solve the problem.

   (a) $\det(BA) = \det(B)\det(A) = -4(3) = -12$.

   (b) $\det(2B) = \det((2I)B) = \det(2I)\det(B) = 2^4(-4) = -64$.

   (c) $\det(A^4) = (\det(A))^4 = 3^4 = 81$.

   (d) $\det(B^T B^{-1}) = \det(B^T)(\det(B))^{-1} = (-4)/(-4) = 1$.

   (e) $\det((A^T B)^{-1} A^T) = \det(B^{-1}(A^T)^{-1}A^T) = \det(A^{-1}) = (\det(A))^{-1} = 1/3$.

2. By the invertible matrix theorem, the given vectors are linearly independent if and only if the determinant

$$\Delta = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{vmatrix}$$

is non-zero. There are many ways to evaluate the determinant. Since there is no obvious column or row full of zeros in which to expand the determinant, we try row reduction. Adding 1 times row 1 to row 3 and -3 times row 1 to row 4 gives

$$\Delta = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 1 & 5 & 5 \\ 0 & 2 & 7 & 3 \end{vmatrix}.$$

Expanding in the first column gives

$$\Delta = \begin{vmatrix} 1 & 5 & 4 \\ 1 & 5 & 5 \\ 2 & 7 & 3 \end{vmatrix}.$$

Adding -1 times row 1 to row 2 and -2 times row 1 to row 3 gives

$$\Delta = \begin{vmatrix} 1 & 5 & 4 \\ 0 & 0 & 1 \\ 0 & -3 & -5 \end{vmatrix}.$$

We could expand in the first column or second row. Since I prefer odd numbered columns or rows, the first row it is. We have

$$\Delta = \begin{vmatrix} 0 & 1 \\ -3 & -5 \end{vmatrix} = 3.$$

Since the determinant is non-zero, the vectors are linearly independent.