MATH122 200610 Quiz 4C Solutions DRAFT

Edward Doolittle

Wednesday, March 29, 2006

1. To evaluate these expressions we use the formulas $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$. For the last question the identities $(AB)^{-1} = B^{-1}A^{-1}$ (note the reversal of order) and $(B^T)^{-1} = (B^{-1})^T$ may also come in handy, depending on how you solve the problem.

   (a) $\det(BA) = \det(B)\det(A) = 5(-2) = -10$.

   (b) $\det(3A) = \det((3I)A) = \det(3I)\det(A) = 3^3(-2) = -54$.

   (c) $\det(A^4) = (\det(A))^4 = (-2)^4 = 16$.

   (d) $\det(A^2B^T) = (\det(A^2))(\det(B^T)) = (\det(A))^2(\det(B)) = 4(5) = 20$.

   (e) $\det((B^TA)^{-1}AB^T) = \det(A^{-1}(B^T)^{-1}AB^T) = (\det(A))^{-1}(\det(B^T))^{-1}(\det(A))(\det(B^T)) = 1$ (just reorder the factors to obtain cancellation).

2. By the invertible matrix theorem, the given vectors are linearly independent if and only if the determinant

$$\Delta = \begin{vmatrix} 1 & -3 & 1 & -2 \\ 2 & -5 & -1 & -2 \\ 0 & -4 & 5 & 1 \\ -3 & 10 & -6 & 8 \end{vmatrix}$$

is non-zero. There are many ways to evaluate the determinant. Since there is no obvious column or row full of zeros in which to expand the determinant, we try row reduction. Adding -2 times row 1 to row 2 and 3 times row 1 to row 4 gives

$$\Delta = \begin{vmatrix} 1 & -3 & 1 & -2 \\ 0 & 1 & -3 & 2 \\ 0 & -4 & 5 & 1 \\ 0 & 1 & -3 & 2 \end{vmatrix}.$$

Expanding in the first column gives

$$\Delta = \begin{vmatrix} 1 & -3 & 2 \\ -4 & 5 & 1 \\ 1 & -3 & 2 \end{vmatrix}.$$

Adding 4 times row 1 to row 2 and -1 times row 1 to row 3 gives

$$\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 0 & -7 & 9 \\ 0 & 0 & 0 \end{vmatrix}.$$

Expanding in the third row gives $\Delta = 0$, and we can conclude that the given vectors are linearly dependent.