MATH 127 Sample Final Exam Solutions DRAFT

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1.

2.

3.

4.

5. (a)

   (b)

   (c)

   (d) We need to find a number n such that $P(n,4) = n(n-1)(n-2)(n-3) = 3024$. Guessing $n = 8$ we have $8 \times 7 \times 6 \times 5 = 1680$, too small. Guessing $n = 9$ we have $9 \times 8 \times 7 \times 6 = 3024$. So there were 9 possible managers available.

   (e) The question isn't entirely clear, so there is some freedom to interpret. I would say we want each yurt to have one manager and one assistant manager. Managers can be assigned in $4! = 24$ ways. Assistant managers can be assigned in $4! = 24$ ways. The total number of different assignments is then $24 \times 24 = 576$.

6. (a)

   (b) The three pairs can be arranged in 3! ways. However, each pair has 2 different configurations, for $2 \times 2 \times 2 = 2^3 = 8$ different ways of ordering the shoes in a pair once they have been set down in a row. Altogether, there are $3! \times 2^3 = 48$ ways of arranging the shoes.

   (c)

   (d) The idea in this part is that rotations are equivalent, so that we can only establish the position of the shoes relative to the position of the first shoe. So it doesn't matter how we set down the first shoe; it doesn't count as a choice, because it can be rotated into any position we like. But after we set down the first shoe, we can treat the remaining places as a row into which we place 5 shoes. There are 5! ways of doing so, which is the answer to the question.

   (e) After we place the first pair, there are 2 shoes we need to place in a row, with $2! = 2$ different ways of doing so. After the pairs are positioned on the display, we can rearrange the shoes within the pairs in $2 \times 2 \times 2 = 2^3 = 8$ different ways. Altogether there are $2! \times 2^3 = 2 \times 8 = 16$ different ways of arranging the display.

7. The key idea in this problem is to convert paths to sequences of Es and Ss.

   (a) For example, draw a path that goes three blocks east, then three blocks south, then one block east. The corresponding "word" describing the path is EEESSSE.

   (b) Any word including four Es and three Ss corresponds to an allowable path, and vice versa. There must be four Es because that's how far you travel east; there must be 3 Ss because that's how far you travel south. Consider, for example, ESEESES. The corresponding path goes one block east, then one south, then two east, then one south, then one east, then one south. Sketch it!

(c) The number of paths is equal to the number of allowable words. The number of allowable words is calculated in the same way as the number of distinct rearrangements of a word like BOOKKEEPER in the textbook (see the end of section 6.4). In this case it would be $7!/(3!4!)$.

(d) $8!/(3!5!)$. Why?

(e) $(n+3)!/(3!n!)$. Why?

8.