

MATH 127 Problem Set 2 Solutions DRAFT

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- 1.
2. (a)
- (b)
- (c) The associated augmented matrix is

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 3 & 1 & -2 & 0 & 1 & 0 \\ -5 & -1 & 9 & 0 & 0 & 1 \end{array} \right].$$

Adding -3 times row 1 to row 2, and 5 times row 1 to row 3, gives

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & -1 & -1 & 5 & 0 & 1 \end{array} \right].$$

Adding 1 times row 2 to row 3 gives

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -3 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 & 1 \end{array} \right].$$

We can now divide the third row by 3, but if we are trying to avoid the use of fractions we can multiply the other two rows by 3, like this:

$$\left[\begin{array}{ccc|ccc} 3 & 0 & -6 & 3 & 0 & 0 \\ 0 & 3 & 12 & -9 & 3 & 0 \\ 0 & 0 & 3 & 2 & 1 & 1 \end{array} \right].$$

Now adding -4 times row 3 to row 2, and 2 times row 3 to row 1, gives

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 7 & 2 & 2 \\ 0 & 3 & 0 & -17 & -1 & -4 \\ 0 & 0 & 3 & 2 & 1 & 1 \end{array} \right].$$

We can now divide through by 3 with no difficulty, or we could even express the answer as

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 7 & 2 & 2 \\ 3 & 1 & -2 & -17 & -1 & -4 \\ -5 & -1 & 9 & 2 & 1 & 1 \end{array} \right]^{-1} = \frac{1}{3} \left[\begin{array}{ccc|ccc} 7 & 2 & 2 \\ -17 & -1 & -4 \\ 2 & 1 & 1 \end{array} \right].$$

Checking by matrix multiplication shows that the above inverse is correct.

3. This question is not best expressed as a linear system of equations. It is meant to be answered by a straightforward matrix multiplication. Let r be the number of pounds of Regular coffee we need, h the number of pounds of High Mountain coffee we need, and c the number of pounds of chocolate. Then we can write

$$\begin{bmatrix} r \\ h \\ c \end{bmatrix} = \begin{bmatrix} 0.80 & 0.75 & 0.50 \\ 0.20 & 0.20 & 0.40 \\ 0.00 & 0.05 & 0.10 \end{bmatrix} \begin{bmatrix} 400 \\ 360 \\ 230 \end{bmatrix}.$$

Think about why the above equation makes sense. The answer can be obtained by matrix multiplication. No row reduction is required. The answer is $r = 705$, $h = 244$, $c = 41$. As a simple check, note that the number of pounds of the result is $400 + 360 + 230 = 990$, and the number of pounds of ingredients is $705 + 244 + 41 = 990$; the two numbers should agree (that is the chemical principle of conservation of mass!).

- 4.