

# MATH 127 Quiz 1 Solutions DRAFT

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1. Let  $x$  be the number of orders of burgers consumed,  $y$  the number of orders of fries consumed, and  $z$  the number of orders of drinks consumed. Then the total number of calories consumed is  $710x + 360y + 230z$ , and the associated equation is  $710x + 360y + 230z = 9360$ . Similarly, the equations for fat and sugar are  $45x + 18y + 0z = 477$  and  $9x + 1y + 56z = 352$  respectively. In summary, the equations which reflect the above information are

$$\begin{array}{rclcl} 710x & + & 360y & + & 230z & = & 9360 \\ 45x & + & 18y & & & = & 477 \\ 9x & + & y & + & 56z & = & 352. \end{array}$$

Of course, “+0z” could appear instead of the blank space in the second equation, and “1y” could appear instead of “y” in the third equation.

2. We can write the system in augmented matrix form as

$$\left[ \begin{array}{ccc|c} 710 & 360 & 230 & 9360 \\ 45 & 18 & 0 & 477 \\ 9 & 1 & 56 & 352 \end{array} \right].$$

A little bit of creativity helps to solve this problem quickly. First note that we can take a factor of 10 from the first row and a factor of 9 from the second row to obtain

$$\left[ \begin{array}{ccc|c} 71 & 36 & 23 & 936 \\ 5 & 2 & 0 & 53 \\ 9 & 1 & 56 & 352 \end{array} \right].$$

Adding  $-14$  times row 2 to row 1 gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 5 & 2 & 0 & 53 \\ 9 & 1 & 56 & 352 \end{array} \right].$$

Adding  $-5$  times row 1 to row 2, and  $-9$  times row 1 to row 3, gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & -38 & -115 & -917 \\ 0 & -71 & -151 & -1394 \end{array} \right].$$

Adding  $-2$  times row 2 to row 3 gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & -38 & -115 & -917 \\ 0 & 5 & 79 & 440 \end{array} \right].$$

Adding 8 times row 3 to row 2 gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & 2 & 517 & 2360 \\ 0 & 5 & 79 & 440 \end{array} \right].$$

Adding  $-2$  times row 2 to row 3 gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & 2 & 517 & 2360 \\ 0 & 1 & -955 & -4766 \end{array} \right].$$

Adding  $-2$  times row 3 to row 2, and then swapping rows 2 and 3, gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & 1 & -955 & -4766 \\ 0 & 0 & 2427 & 12135 \end{array} \right].$$

Now divide row 3 by 2427 to obtain

$$\left[ \begin{array}{ccc|c} 1 & 8 & 23 & 194 \\ 0 & 1 & -955 & -4766 \\ 0 & 0 & 1 & 5 \end{array} \right].$$

Now we could solve the system by back substitution, but I prefer continuing with row reduction. Adding 955 times row 3 to row 2, and  $-23$  times row 3 to row 1, gives

$$\left[ \begin{array}{ccc|c} 1 & 8 & 0 & 79 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{array} \right].$$

Finally, adding  $-8$  times row 2 to row 1 gives

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{array} \right].$$

So it seems that our boys ordered 7 burgers, 9 orders of fries, and only 5 drinks.

Of course, there are many other ways to carry out the above row reduction. I suggest you try to avoid fractions, but if you use a calculator and decimal fractions you may be able to solve the problem in a quite direct manner, following the examples in the textbook. Since decimal fractions on a calculator are just an approximation, you should check your final answer to make sure that it really does satisfy the conditions of the problem.

You might also tackle the problem in completely different ways. You might, for example, use the information that each of  $x$ ,  $y$ , and  $z$  is a small nonnegative integer to help you guess a solution. You should remember to check whether your guess is correct, and if it isn't, to guess again.

In retrospect, this problem is too hard for a test question. Too much arithmetic is required, which obscures the underlying concepts.